1. **Antiderivacije**

1. Pronađi tri antiderivacije funkcije \( f(x) = x^2 \).
2. Odredi sve antiderivacije funkcije \( f(x) = x^2 \).
3. Pronađi dvije antiderivacije funkcije \( f(x) = \cos x \).
4. Pronađi antiderivaciju \( F(x) \) funkcije \( f(x) = x^3 \) za koju je \( F(1) = 1 \).
5. Pronađi onu antiderivaciju \( F(x) \) funkcije \( f(x) = x^2 - x \) za koju vrijedi \( F(1) = 2 \).
6. Pronađi antiderivaciju \( F(x) \) funkcije \( f(x) = \frac{1}{1+x^2} \) koja zadovoljava uvjet \( F(0) = 1 \).
7. Ima li funkcija \( f(x) = e^x \) antiderivaciju \( F(x) \) za koju je \( F(0) = -10 \)?
8. Odredi bar jednu antiderivaciju funkcije \( f(x) = 2e^x - 3x + 5 \).
9. Odredi bar jednu antiderivaciju funkcije \( f(x) = \sin 2x \).
10. Uz pomoć jednog trigonometrijskog identiteta pronadite antiderivaciju funkcije \( f(x) = \sin^2 x \).
11. Je li funkcija \( F(x) = \sqrt{x^2 - 4x + 1} \) antiderivacija funkcije \( f(x) = \frac{x - 2}{\sqrt{x^2 - 4x + 1}} \)?
12. Je li funkcija \( F(x) = x^3 - 2x^2 - 3 \) antiderivacija funkcije \( f(x) = 3x^2 - 4x - 3 \)?
13. Je li funkcija \( F(x) = 3 \sin x - \sin^3 x \) antiderivacija funkcije \( f(x) = 3 \cos^3 x \)?
2. Integriranje pomoću tablice i osnovnih pravila

Služeći se tablicom i osnovnim pravilima pronadi neodređene integrale

14. $\int x^3 \, dx$  
15. $\int \sqrt[3]{x} \, dx$  
16. $\int \sqrt[4]{x^5} \, dx$

17. $\int \frac{1}{x^4} \, dx$  
18. $\int \frac{t^2}{\sqrt{t}} \, dt$  
19. $\int \frac{x^{3/5}}{\sqrt{x^2}} \, dx$

20. $\int 6dx$  
21. $\int 7t \, dt$  
22. $\int \sqrt{2x} \, dx$

23. $\int (3x^2 + 2\sin x) \, dx$  
24. $\int (s - 4\cos s) \, ds$  
25. $\int (5x - 1)^2 \, dx$

26. $\int \frac{4x-1}{\sqrt{x}} \, dx$  
27. $\int \frac{s+s^2}{s^3} \, ds$  
28. $\int \frac{1-2x}{\sqrt[3]{x}} \, dx$

29. $\int \frac{(x-3)^2}{x} \, dx$  
30. $\int \frac{(x^2+1)^3}{x^2} \, dx$  
31. $\int \frac{(2-x)^4}{x^3} \, dx$

32. $\int 3^x \, dx$  
33. $\int 4^{x+2} \, dx$  
34. $\int (1 - 2e^x) \, dx$

35. $\int \frac{1}{x^2+9} \, dx$  
36. $\int \frac{1}{x^2-2} \, dx$  
37. $\int \frac{1}{3-x^2} \, dx$

38. $\int \frac{1}{\sqrt{x^2+4}} \, dx$  
39. $\int \frac{1}{\sqrt{x^2-4}} \, dx$  
40. $\int \frac{1}{\sqrt{4-x^2}} \, dx$

41. $\int \sqrt{x^2 + 16} \, dx$  
42. $\int \sqrt{x^2 - 16} \, dx$  
43. $\int \sqrt{16 - x^2} \, dx$
3. Metoda zamjene

Pogodnim zamjenama odredi integrale

44. \( \int (5x + 2)^6 \, dx \) 45. \( \int \sqrt{4 - 3x} \, dx \) 46. \( \int x^3 \sqrt{x^2 - 6} \, dx \)

47. \( \int \cos(2x - 1) \, dx \) 48. \( \int x \sin(x^2) \, dx \) 49. \( \int \sin x \ \cos^2 x \, dx \)

50. \( \int \frac{x}{\sqrt{x^2 - 9}} \, dx \) 51. \( \int \frac{\cos t}{\sin^3 t} \, dt \) 52. \( \int \frac{1}{x \ln x} \, dx \)

53. \( \int \frac{e^{2x}}{e^x + 2} \, dx \) 54. \( \int \frac{t^2}{t^3 + 1} \, dt \) 55. \( \int \frac{\ln x}{x} \, dx \)

56. \( \int x^2 e^{x^3 - 3} \, dx \) 57. \( \int x^3 (1 - x^4)^{-2} \, dx \) 58. \( \int x^3 \sqrt{3 - x^2} \, dx \)

Riješi integrale tako da kvadratni izraz prvo predociš kao zbroj ili razliku kvadrata, a zatim uvedeš zamjenu

59. \( \int \frac{1}{x^2 + 4x + 8} \, dx \) 60. \( \int \frac{1}{7 - 6x - x^2} \, dx \)

61. \( \int \frac{1}{\sqrt{x^2 + 8x}} \, dx \) 62. \( \int \frac{1}{\sqrt{3 + 2x - x^2}} \, dx \)

63. \( \int \sqrt{x^2 + 2x - 1} \, dx \) 64. \( \int \sqrt{2x - x^2} \, dx \)

4. Metoda djelomične integracije

Djelomičnim integriranjem odredi integrale

65. \( \int \ln x \, dx \) 66. \( \int xe^x \, dx \) 67. \( \int x \sin x \, dx \)

68. \( \int x^2 \cos x \, dx \) 69. \( \int x^2 2^x \, dx \) 70. \( \int x^4 \log x \, dx \)

71. \( \int \frac{\ln x}{\sqrt{x}} \, dx \) 72. \( \int \frac{x^2 + x}{e^x} \, dx \) 73. \( \int \frac{x}{\cos^2 x} \, dx \)

74. \( \int \arctg \, dt \) 75. \( \int x \arcsin x \, dx \) 76. \( \int \log^2 t \, dt \)

Dvostrukom primjenom formule za djelomičnu integraciju zadani integral svedi na integralnu jednadžbu, a potom ju riješi

77. \( \int e^x \sin x \, dx \) 78. \( \int 3^x \cos x \, dx \) 79. \( \int e^{-x} \cos x \, dx \)
5. Integriranje racionalnih funkcija

Odredi integrale djelomičnih razlomaka

80. \( \int \frac{3}{x} \, dx \)
81. \( \int \frac{-1}{x^6} \, dx \)
82. \( \int \frac{5}{2x-3} \, dx \)
83. \( \int \frac{-2}{(1-3x)^3} \, dx \)
84. \( \int \frac{1}{x^2-2x+3} \, dx \)
85. \( \int \frac{1}{x^2+4x+5} \, dx \)
86. \( \int \frac{-2x+4}{x^2+3} \, dx \)
87. \( \int \frac{x+2}{-x^2+2x-5} \, dx \)
88. \( \int \frac{x}{(x^2+1)^2} \, dx \)
89. \( \int \frac{1}{(x^2+1)^2} \, dx \)

Odredi integrale pravih racionalnih funkcija

90. \( \int \frac{6x}{x^2+4x-5} \, dx \)
91. \( \int \frac{x-5}{x^2+2x-3} \, dx \)
92. \( \int \frac{3x^2+1}{(x+4)(x-3)^2} \, dx \)
93. \( \int \frac{-x}{(x-2)^3} \, dx \)
94. \( \int \frac{x^2-3x+6}{(x+1)(x^2+9)} \, dx \)
95. \( \int \frac{1}{x^3+x^2} \, dx \)
96. \( \int \frac{7x-2}{x^3-4x} \, dx \)
97. \( \int \frac{6}{x^3+2x^2-8x} \, dx \)
98. \( \int \frac{1}{x^4-1} \, dx \)
99. \( \int \frac{x^3}{(x^2+1)^2} \, dx \)

Odredi integrale racionalnih funkcija

100. \( \int \frac{x^4-9x^2-1}{x+3} \, dx \)
101. \( \int \frac{2x^4-5x^2}{x^2-4} \, dx \)
102. \( \int \frac{x^3+x^2+1}{x^2+x-2} \, dx \)
103. \( \int \frac{x^2}{x^2+16} \, dx \)
104. \( \int \frac{2x^3-17x}{x^2+3x-4} \, dx \)
105. \( \int \frac{x^3+4x^2+x}{x^3-1} \, dx \)
106. \( \int \frac{x^4-x^3-16}{x^3+4x} \, dx \)
107. \( \int \frac{x^3-3x+4}{x^3-2x^2+2x} \, dx \)
108. \( \int \frac{x^6}{x^4-1} \, dx \)
109. \( \int \frac{x^5}{(x^2+1)^2} \, dx \)
6. Integriranje funkcija s korijenom

Pogodnim zamjenama zadane integrale svedi na integrale racionalnih funkcija i riješi ih

110. \( \int \frac{1}{\sqrt{x+1}} \, dx \)
111. \( \int \sqrt{\frac{x-1}{x}} \, dx \)
112. \( \int \frac{x}{\sqrt{(x+1)^2}} \, dx \)
113. \( \int \frac{\sqrt{x}}{x(\sqrt{x}-1)} \, dx \)
114. \( \int \frac{1}{x^2 \sqrt{\frac{x+2}{x}}} \, dx \)
115. \( \int \frac{1}{x} \sqrt{\frac{x+2}{x}} \, dx \)
116. \( \int \frac{1}{\sqrt{x^2-1}} \, dx \)
117. \( \int \frac{\sqrt{1-x^2}}{x} \, dx \)

7. Integriranje trigonometrijskih funkcija

Uz pomoć formula koje umnožak sinusa i kosinusa pretvaraju u zbroj ili razliku riješi integrale

118. \( \int \sin 2x \ \sin 4x \, dx \)
119. \( \int \sin x \ \cos 3x \, dx \)
120. \( \int \cos 3x \ \cos 4x \, dx \)
121. \( \int \cos^2 5x \, dx \)
122. \( \int \sin x \ \sin 2x \ \sin 3x \, dx \)
123. \( \int \sin^2 2x \ \cos x \, dx \)

Uz pomoć neke od zamjena \( t = \sin x \), \( t = \cos x \), \( t = \tan x \) ili \( t = \tan \frac{x}{2} \) riješi integrale

124. \( \int \frac{\sin x}{\cos^2 x} \, dx \)
125. \( \int \frac{\cos x}{\sin^3 x} \, dx \)
126. \( \int \frac{3 \sin x}{(\cos x+3)^4} \, dx \)
127. \( \int \frac{\cos x}{5-\cos^2 x} \, dx \)
128. \( \int \frac{\sin^2 x}{\cos^4 x} \, dx \)
129. \( \int \frac{1-4 \cos^2 x}{\sin^4 x} \, dx \)
130. \( \int \frac{1}{\sin x \ \cos x} \, dx \)
131. \( \int \frac{1}{3 \sin^2 x+1} \, dx \)
132. \( \int \frac{1}{\sin x+1} \, dx \)
133. \( \int \frac{1}{\cos x} \, dx \)
134. \( \int \frac{\cos x}{\cos x+1} \, dx \)
135. \( \int \frac{1}{3 \sin x-4 \cos x} \, dx \)
8. Različiti zadatci

Riješi integrale

136. \(\int \frac{3}{\sqrt[3]{x} \sqrt{x}} \, dx\) 137. \(\int 2^x + 2 \, 3^{-x-3} \, dx\)

138. \(\int \frac{1}{\sqrt[3]{3x-3}} \, dx\) 139. \(\int \frac{x^2}{\sqrt[3]{4x^4-8}} \, dx\)

140. \(\int \frac{1}{5-4x-x^2} \, dx\) 141. \(\int \frac{2x-1}{x^2-2x+2} \, dx\)

142. \(\int \frac{3x-4}{x^3+x} \, dx\) 143. \(\int \frac{x^3-x+2}{x^3-x} \, dx\)

144. \(\int (3x^2 + 1) \arctan x \, dx\) 145. \(\int \arcsin 2x \, dx\)

146. \(\int x \log 4x \, dx\) 147. \(\int 2^x \sin x \, dx\)

148. \(\int \frac{3\sqrt[3]{x}+1}{\sqrt{x}} \, dx\) 149. \(\int \frac{x-1}{\sqrt{x+1}} \, dx\)

150. \(\int \frac{1-\cos x}{\sin^2 x} \, dx\) 151. \(\int \frac{\sin x + \cos x}{\cos^3 x} \, dx\)
ZII. ODREĐENI INTEGRALI

1. Računanje određenog integrala

Služeći se tablicom, osnovnim pravilima i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

152. \( \int_{-1}^{2} x^3 \, dx \)
153. \( \int_{0}^{5} \sqrt{x^7} \, dx \)
154. \( \int_{1}^{e} x^{-1} \, dx \)

155. \( \int_{\pi/2}^{\pi} \cos x \, dx \)
156. \( \int_{0}^{3} \frac{1}{x^2 + 1} \, dx \)
157. \( \int_{-2}^{0} 3^x \, dx \)

158. \( \int_{0}^{4} x \left(1 - \sqrt{x}\right) \, dx \)
159. \( \int_{4}^{0} \left(3 - \sqrt{x}\right)^2 \, dx \)
160. \( \int_{-1}^{1} \left(2 - 3\sqrt{x}\right)^3 \, dx \)

161. \( \int_{1}^{64} \left(4 \sqrt{x} - \frac{4}{x}\right)^2 \, dx \)
162. \( \int_{1}^{16} \left(4 \sqrt{x} - \frac{4}{x}\right)^2 \, dx \)
163. \( \int_{1}^{4} \frac{(\sqrt{x} - x)^3}{x} \, dx \)

Služeći se metodom zamjene i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

164. \( \int_{-2}^{2} \sqrt{2x + 5} \, dx \)
165. \( \int_{-2}^{3} \sqrt{2 - 3x} \, dx \)
166. \( \int_{\pi/4}^{2\pi} \sin^2 x \cdot \cos x \, dx \)

167. \( \int_{-3}^{-2} \frac{1}{(x + 4)^3} \, dx \)
168. \( \int_{-1}^{3} \frac{x^2 - 1}{x + 2} \, dx \)
169. \( \int_{1}^{e} \frac{\ln x}{x} \, dx \)

170. \( \int_{0}^{4} \frac{x}{\sqrt{x + 1}} \, dx \)
171. \( \int_{-4}^{1} \frac{1 + x}{\sqrt{5 - x}} \, dx \)
172. \( \int_{1}^{27} \frac{\sqrt{x}}{x \left(1 + \frac{3}{x}\right)} \, dx \)
Zbirka zadataka

Služeći se metodom djelomične integracije i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

173. \[ \int_0^\pi \cos x \, dx \]
174. \[ \int_0^1 \ln x \, dx \]
175. \[ \int_{-1}^1 x^2 e^x \, dx \]
176. \[ \int_1^{10} \log \frac{x}{x^2} \, dx \]
177. \[ \int_{-1}^0 x \arctan x \, dx \]
178. \[ \int_{-1}^4 \log_2 \frac{x}{\sqrt{x}} \, dx \]

Izračunaj integrale:

179. \[ \int_0^{\pi/2} x \sin 2x \, dx \]
180. \[ \int_{-1}^0 x \ln (x + 2) \, dx \]
181. \[ \int_0^{2/5} \arctan (3x + 5) \, dx \]
182. \[ \int_0^2 e^x \cos x \, dx \]
183. \[ \int_{-2}^{5/3} \frac{x}{x^2 + 3} \, dx \]
184. \[ \int_0^\infty \frac{x^3 - 3x - 2}{x + 1} \, dx \]

Izračunaj integrale tako da prvo provjeriš parnost podintegralne funkcije ili njenih pribrojnika

185. \[ \int_{-1}^1 (x^4 - x^2 + 5) \, dx \]
186. \[ \int_{-3}^3 (x^3 - x \cos x) \, dx \]
187. \[ \int_{-\pi}^\pi (\sin^2 x - 3 \cos x) \, dx \]
188. \[ \int_{-5}^5 (x + \sin x - x^2 \tan x) \, dx \]
189. \[ \int_{-2}^{\sqrt{2}} (x^2 \sin x - 4 \cot x + \sqrt{2 - x}) \, dx \]
190. \[ \int_{-4}^4 x^2 (\sin x + \cos x)^2 \, dx \]

Odredi funkciju \( f(x) \) i izračunaj \( f(x_0) \), ako je

191. \[ f(x) = \int_0^x \sqrt{t} \, dt, \quad x_0 = 4 \]
192. \[ f(x) = \int_{-1}^x (t^2 + 2t) \, dt, \quad x_0 = 0 \]
193. \[ f(x) = \int_{-e}^1 \frac{1}{t} \, dt, \quad x_0 = -e \]
194. \[ f(x) = \int_0^1 (\sqrt{t} - 1) \, dt, \quad x_0 = 1 \]
195. \[ f(x) = \int_{-2}^1 \frac{t^3 - 1}{t^2} \, dt, \quad x_0 = 2 \]
196. \[ f(x) = \int_1^3 \frac{t}{t + 1} \, dt, \quad x_0 = 3 \]
2. Površina ravninskog lika

Izračunaj površinu lika omeđenog krivuljama

197. \( x = 3, \ y = 0, \ y = x^2 \)
199. \( x = -1, \ x = 3, \ y = 0, \ y = 3x^2 - 2x + 1 \)
201. \( y = 0, \ y = \sin x \ za \ 0 \leq x \leq \pi \)
203. \( x = 1, \ x = 3, \ y = 1 - 2x, \ y = x^2 - 2x + 3 \)
205. \( x = 0, \ y = x^2, \ y = (x - 4)^2 \)
207. \( x = y + 2, \ x = y^2 \)
209. \( y = x^2 - 3x + 2, \ y = -x^2 + 3x - 2 \)
211. \( x = y^2 - 5y + 6, \ x = -y^2 + 7y - 4 \)
213. \( x = 0, \ y = 0, \ y = x^3 - 1 \)
215. \( y = 2 - x, \ y = -\sqrt{x}, \ y = x^3 \)
217. \( y = x^2 - x^3, \ tangenta \ u \ točki \ T(1,0) \)
U narednim zadacima površinu lika omeđenog zadanim krivuljama izračunaj na dva načina:
integriranjem funkcija $y(x)$ po $x$
integriranjem funkcija $x(y)$ po $y$

218. $x = y^2, \ y = x^2$
219. $y = x^2, \ y = x^3$
220. $y^2 = x + 3, \ y^2 = 4x$
221. $y = 0, \ y = \sqrt{x+2}, \ y = \sqrt{2x}$
222. $y = 0, \ y = -x+6, \ y = \sqrt{x}$
223. $2x+3y = 10, \ xy = x+1$
224. $x = 0, \ x = 1, \ x = \tan y, \ y = \pi / 2$
225. $x = 0, \ y = 0, \ y = 1, \ y = \ln x$

3. Obujam rotacijskog tijela

Izračunaj obujam tijela nastalog vrtnjom, oko osi $x$, lika omeđenog krivuljama

226. $x = 2, \ y = 0, \ y = x^2$
227. $y = 0, \ y = x^2 - 5x$
228. $y = 4, \ y = x^2$
229. $y = x, \ y = x^2 - x$
230. $y = x+4, \ y = x^2 + 2$
231. $x = 0, \ x = 1 - y^2$
232. $y = 0, 3y = 4-x, \ y = \sqrt{x}$
233. $xy = 1, 2y = 1, \ y = \sqrt{x}$
234. $y = x-1, \ y^2 = 2x+1$
235. $y = x^2 - 2x, \ y = -x^2 + 2x + 6$

Izračunaj obujam tijela nastalog vrtnjom, oko osi $y$, lika omeđenog krivuljama

236. $x = 0, \ x = 1 - y^2$
237. $xy = 1, \ y = 1, \ y = 2$
238. $x = 2, \ y = 0, \ y = x^2$
239. $x = y^2, \ y = x^2$
240. $y = 0, 3y = 4-x, \ y = \sqrt{x}$
241. $xy = 1, 8y = x^2, \ y = x^3$
4. Duljina luka ravninske krivulje

Izračunaj duljinu luka krivulje

\[ y = \frac{2}{3} \sqrt{x^3} \quad \text{za} \quad 3 \leq x \leq 8 \]

\[ x = \sqrt{y^3} \quad \text{za} \quad 0 \leq y \leq \frac{5}{9} \]

\[ y^2 = 4x \quad \text{između točaka} \quad A(0,0) \text{ i } B(1,2) \]

\[ y = \frac{1}{2} x^2 - x \quad \text{između točaka} \quad A \left(0, -\frac{1}{2}\right) \text{ i } B \left(3, \frac{3}{2}\right) \]

\[ y = \ln \left(1 - x^2\right) \quad \text{za} \quad 0 \leq x \leq \frac{1}{2} \]

\[ y = 2e^{\frac{x}{2}} \quad \text{za} \quad \ln 24 \leq x \leq \ln 48 \]

\[ x = \ln \sin y \quad \text{za} \quad \frac{\pi}{3} \leq y \leq \frac{\pi}{2} \]

\[ y = \arcsin e^x \quad \text{za} \quad \ln \frac{\sqrt{3}}{2} \leq x \leq \ln \frac{2\sqrt{2}}{3} \]

5. Površina rotacijske plohe

Izračunaj površinu plohe nastale vrtnjom, oko osi \(x\) , luka krivulje

\[ y = \frac{1}{3} x^3 \quad \text{za} \quad 0 \leq x \leq \sqrt{3} \]

\[ y^2 = 4x \quad \text{između točaka} \quad A \left(3, -2\sqrt{3}\right) \text{ i } B \left(3, 2\sqrt{3} \right) \]

\[ x^3 + y^3 = 1 \quad \text{za} \quad 0 \leq x \leq 1 \]

\[ y = \frac{2}{\sin x} \quad \text{za} \quad \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \]

Izračunaj površinu plohe nastale vrtnjom, oko osi \(y\) , luka krivulje

\[ x = \sqrt{1 - y^2} \quad \text{za} \quad 0 \leq y \leq 1 \]

\[ y = x^2 \quad \text{između točaka} \quad A \left(-\sqrt{2}, 2\right) \text{ i } B \left(\sqrt{2}, 2\right) \]

\[ x = \frac{1}{3} \sqrt{y^3} \quad \text{za} \quad 0 \leq y \leq 2 \]
6. Numerička integracija

Trapeznom formulom, uz zadani korak $h$, izračunaj približnu vrijednost određenih integrala

257. $\int_{2}^{3} \sin \left(x^2 + 1\right) dx$, $h = 0.2$
258. $\int_{3}^{5} x^2 \cos x dx$, $h = 0.4$
259. $\int_{1}^{1.5} \ln^2 \left(x^3 + 10\right) dx$, $h = 0.1$
260. $\int_{-3}^{-2} \sqrt{-x^2 - 5x - 6} dx$, $h = 0.25$
261. $\int_{1}^{2} \frac{x^2 - 15}{\log x} dx$, $h = 0.2$
262. $\int_{10}^{12} \frac{x^3 + 3}{x + 1} dx$, $h = 0.4$
263. $\int_{0.1}^{0.7} (x - 3 \arctan x) dx$, $h = 0.1$
264. $\int_{-1}^{0} \arcsin \left(x + e^x\right) dx$, $h = 0.2$

Simpsonovom formulom, uz zadani korak $h$, izračunaj približnu vrijednost određenih integrala

265. $\int_{0.1}^{0.5} \frac{1}{\cos x} dx$, $h = 0.1$
266. $\int_{-1}^{0} \frac{1}{\cos x} dx$, $h = 0.2$
267. $\int_{1}^{2} e^x \ln x dx$, $h = 0.25$
268. $\int_{0}^{11} x^2 \log x dx$, $h = 0.5$
269. $\int_{2}^{6} \frac{x^3 - 2}{x + 2} dx$, $h = 0.25$
270. $\int_{3}^{4.2} \frac{3x + 3}{x^2 + 2} dx$, $h = 0.2$
271. $\int_{0}^{0.8} \sqrt{x - x^2} dx$, $h = 0.1$
272. $\int_{-2}^{0} \frac{1}{\sqrt{x + e^x}} dx$, $h = 0.5$
273. $\int_{-1}^{1} x \tan^2 x dx$, $h = 0.5$
274. $\int_{2}^{3} 4 \cot x dx$, $h = 0.25$
275. $\int_{0.5}^{1.1} \frac{\sqrt{x + \ln x}}{\cos x} dx$, $h = 0.1$
276. $\int_{4}^{5} \frac{\sin x - x}{\arctan x} dx$, $h = 0.25$
7. Različiti zadatci

Izračunaj vrijednost integrala:

\[
\begin{align*}
277. & \quad \int_{0}^{4} x \sqrt{x} \, dx \\
278. & \quad \int_{0}^{6} \left( \sqrt{x} - \sqrt[3]{x} \right) \, dx \\
279. & \quad \int_{-1}^{2} x \sqrt{1 - x^2} \, dx \\
280. & \quad \int_{0}^{2} \left( x - 1 \right) \sqrt{2x - x^2} \, dx \\
281. & \quad \int_{-3}^{2} \left| x \right| \, dx \\
282. & \quad \int_{-1}^{4} \left| x - 3 \right| \, dx \\
283. & \quad \int_{1}^{2} \sqrt{x^2 - 2x + 1} \, dx \\
284. & \quad \int_{2}^{1} \sqrt{x^2 - 2x + 1} \, dx \\
285. & \quad \int_{-3}^{2} \sqrt{x^2 + 4x + 4} \, dx \\
286. & \quad \int_{2}^{-1} \sqrt{4x^2 - 4x + 1} \, dx \\
287. & \quad \int_{\pi}^{2\pi} \sqrt{1 - \cos^2 x} \, dx \\
288. & \quad \int_{0}^{\pi} \sqrt{1 - \sin^2 x} \cdot \sin 2x \, dx \\
289. & \quad \int_{-1}^{3} e^{|x|} \, dx \\
290. & \quad \int_{3}^{1} \left| x^2 - 3x + 2 \right| \, dx
\end{align*}
\]

Uz pomoć integralnog računa izvedi formule za:

291. površinu i opseg kruga
292. obujam i površinu uspravnog kružnog stožca
293. obujam i površinu kugle
294. obujam i površinu torusa
ZIII. DIFERENCIJALNE JEDNADŽBE

1. Provjera rješenja

Provjeri jesu li funkcije \( y \) rješenja diferencijalnih jednadžbi

295. \( y' + y = 2 \cos x \), \( y = \sin x + \cos x \)

296. \( xy' + y = 0 \), \( y = \frac{1}{x} + \frac{1}{x^2} \)

297. \( y' - 2y = 2x - 1 \), \( y = Ce^{2x} - x \)

298. \( y'tgx - 3y + 2 = 0 \), \( y = C \sin^3 x + 1 \)

299. \( y'' + y + 2 \sin x = 0 \), \( y = x \cos x \)

300. \( y'' - 2y' + y = e^x \), \( y = x^2e^x \)

301. \( y' \cos x - 2y' \sin x = 0 \), \( y = \tan x \)

302. \( y'' + y = x^2 + 2 \), \( y = C_1 \sin x + C_2 \cos x + x^2 \)

303. \( 4x^2y''' + y' = 0 \), \( y = \sqrt{x^3} \)

304. \( x^3y''' + x^2y''' + xy' = 3 \), \( y = \frac{3}{2} \ln x \)

305. \( y''' + y'' + y = \cos x \), \( y = \sin x \)

306. \( y''' - 5y'' + 6y' = 0 \), \( y = C_1 + C_2e^{2x} + C_3e^{3x} \)
2. Diferencijalne jednadžbe koje se rješavaju neposrednim integriranjem

Neposrednim integriranjem odredi opće rješenje diferencijalnih jednadžbi

307. \( y' = 2x - \cos x \)
308. \( y' + e^x = 1 \)
309. \( xy' = 1 \)
310. \( xy'' = 2 \ln x + 1 \)
311. \( y'' = 6x + 2 \)
312. \( y'' + x^{-2} = x \)
313. \( x^2 y'' = x^2 e^x + 1 \)
314. \( y'' = \cos x - x \sin x \)
315. \( y''' = 0 \)
316. \( y''' - 3 = \cos x \)
317. \( y''' + (x + 3)e^x = 0 \)
318. \( x^3 y''' = 4 \ln x \)

Pronađi pojedinačno rješenje diferencijalnih jednadžbi koje zadovoljava zadane uvjete

319. \( y' = 3x^2 + 2x - \frac{1}{x} \) : \( y(1) = 3 \)
320. \( y' = \cos x - \sin x \) : \( y(\pi) = 2y(0) \)
321. \( y' = 5e^{3x} - 10x \) : \( y(0) = y'(0) \)
322. \( y'' = 12x^2 - 2 \) : \( y(1) = 1, y(2) = 2 \)
323. \( y'' = \frac{2x - 6}{x^4} \) : \( y(1) = y(2), y(4) = 0 \)
324. \( y'' = \frac{2 \sin x}{\cos^2 x} + \frac{2 \cos x}{\sin^3 x} \) : \( y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}, y'\left(\frac{\pi}{4}\right) = 1 \)
325. \( y''' = e^x \) : \( y(0) = 1, y'(0) = 1, y''(0) = -1 \)
326. \( y''' = 6 - \frac{60}{x^6} \) : \( y(1) = 3, y'(1) = 8, y''(2) = \frac{3}{8} \)
3. Diferencijalne jednadžbe prvog reda

3.1. Diferencijalna jednadžba s razdvojenim promjenljivim

Riješi diferencijalne jednadžbe

327. \( \sqrt{x}dx = y^3 dy \) \hspace{1cm} 328. \( \sqrt{y^2 - 1}dx = ydy \)

329. \( \ln xdx - xdy = 0 \) \hspace{1cm} 330. \( \frac{dx}{y^2} + \frac{dy}{x} = 0 \)

331. \( (x + \sin x)dx = (y + \cos y)dy \) \hspace{1cm} 332. \( xe^y dx(y + 1)e^y dy \)

Riješi diferencijalne jednadžbe tako da prvo razdvojiš diferencijale i promjenljive

333. \( x^2y' = y^2 \) \hspace{1cm} 334. \( y' = 2xy \)

335. \( x + yy' = 0 \) \hspace{1cm} 336. \( y' = \sqrt{xy} \)

337. \( 2yy' = y^2 + 3 \) \hspace{1cm} 338. \( e^{x+y}y' = 1 \)
3.2. Homogena diferencijalna jednadžba

Zamjenom $z = \frac{x}{y}$, a potom razdvajanjem promjenljivih $x$ i $z$, riješi diferencijalne jednadžbe

339. \[ y' = \frac{y - x}{x} \]

340. \[ y' = \frac{(x^2 - y^2)y}{x^3} \]

341. \[ y' = \frac{y}{x - y} \]

342. \[ y' \ln \frac{y}{x} = \frac{y}{x} (\ln \frac{x}{y} - 1) \]

Zamjenom $z = \frac{y}{x}$, a potom razdvajanjem promjenljivih $x$ i $z$, riješi diferencijalne jednadžbe

343. \[ y' = \frac{x + y}{x} \]

344. \[ y' = \frac{xy + y}{x^2} \]

345. \[ y' = \frac{x^2 + y^2}{xy} \]

346. \[ y' \sin \frac{y}{x} = \frac{y}{x} \sin \frac{x}{y} + 1 \]

Riješi diferencijalne jednadžbe

347. \[ (x + y)y' = y \]

348. \[ x^3 y' = (x^2 + y^2)y \]

349. \[ y' = \frac{y}{x} + \cos^2 \frac{y}{x} \]

350. \[ y' = \frac{y}{x} + \sin \frac{y}{x} \]
3.3. Linearna diferencijalna jednadžba

Odredi opće rješenje homogenih linearnih diferencijalnih jednadžbi

351. \[ y' + 3y = 0 \]

352. \[ y' - \frac{1}{x} y = 0 \]

353. \[ y' + (\sin x)y = 0 \]

354. \[ y' - (\ln x + 1)y = 0 \]

Pronađi opće rješenje diferencijalnih jednadžbi tako da prvo riješiš njihove homogene jednadžbe, a zatim primijeniš metodu varijacije konstante

355. \[ y' - y = 2xe^x \]

356. \[ y' + 2y = 4x^2 \]

357. \[ y' + \frac{1}{x} y = 3x + 2 \]

358. \[ y' + \frac{1}{x} y = \cos x \]

359. \[ y' - 2xy = e^{x^2} \]

360. \[ y' + (\sin x)y = \sin x \]

361. \[ xy' + y = 5x^4 \]

362. \[ xy' - y = 2x \ln x \]

363. \[ x^2y' + y = x^2 e^{\frac{1}{x}} \]

364. \[ y' \cos^2 x - y - 1 = 0 \]

Pronađi pojedinačno rješenje diferencijalnih jednadžbi koje zadovoljava zadani uvjet

365. \[ y' - y = -1 \] : \[ y(0) = 5 \]

366. \[ y' + \frac{1}{x} y = x \] : \[ y(1) = 1 \]

367. \[ y' - (\cos x)y = 0 \] : \[ y'(0) = 1 \]

368. \[ y' - \frac{1}{x} y = \frac{1}{x} \] : \[ y'(2) = 0 \]
3.4. Bernoullieva diferencijalna jednadžba

Pronađi opće rješenje diferencijalnih jednadžbi tako da prvo riješiš njihove homogene jednadžbe, a zatim primijeniš metodu varijacije konstante $C$ te razdvojiš promjenljive $x$ i $C$.

369. $y' - y = y^2$
370. $xy' - y = 2x\sqrt{xy}$
371. $y' - 2y = \frac{e^{4x}}{2y}$
372. $y' + \frac{1}{x}y = \frac{x}{y^2}$
373. $y' + 2xy - 2xy^3 = 0$
374. $y' - y + \sqrt{y^2} = 0$

4. Diferencijalne jednadžbe drugog reda

4.1. Linearna diferencijalna jednadžba

Snižavanjem reda riješi linearne diferencijalne jednadžbe

375. $y'' - 2y' = e^x$
376. $xy'' - y' = 3x^2$
377. $x^2y'' + xy' = 6\ln x$
378. $y'' \sin x - y' \cos x = 2\sin^3 x$
4.2. Linearna diferencijalna jednadžba s konstantnim koeficijentima

Odredi opće rješenje homogenih linearnih diferencijalnih jednadžbi

379. \( y'' + y' - 6y = 0 \)  
380. \( y'' - 4y' = 0 \)

381. \( \frac{1}{3} y'' - 2y' + 3y = 0 \)  
382. \( y'' - y' + \frac{1}{4} y = 0 \)

383. \( y'' + 25y = 0 \)  
384. \( y'' + 4y' + 13y = 0 \)

Pronađi opće rješenje diferencijalnih jednadžbi tako da prvo riješiš nijhove homogene jednadžbe, a zatim primijeniš metodu varijacije konstanti

385. \( y'' - 2y' + y = 2e^x \)  
386. \( y'' + y = x \)

387. \( y'' + 2y' = e^{-2x} \)  
388. \( y'' - 4y' + 3y = 8xe^{3x} \)

389. \( y'' + y = \sin x \)  
390. \( y'' - 3y' + 2y = 3\sin x + \cos x \)

391. \( y'' + 4y = -4\sin 2x - 4\cos 2x \)  
392. \( y'' - y' = e^x \sin x \)

393. \( 2y'' - 2y = (2x - 1)e^x \)  
394. \( (e^x + 1)(y'' + y') = e^x \)

Pronađi pojedinačno rješenje diferencijalnih jednadžbi koje zadovoljava zadane uvjete

395. \( y'' + y' = 2x - 1 \)  
: \( y(0) = 4 \), \( y(1) = 2 \)

396. \( y'' - 2y' + y = e^{2x} \)  
: \( y(0) = 0 \), \( y'(0) = 1 \)

397. \( y'' - 6y' + 9y = -e^{3x} \sin x \)  
: \( y'(0) = 2 \), \( y'(\pi) = 0 \)

398. \( y'' + y = 1 \)  
: \( y'(0) = 1 \), \( y''(0) = 2 \)
5. Različiti zadatci

Riješi diferencijalne jednadžbe

399. \( x^3 y''' = 1 \)  
400. \( \ln(y'') = x \)

401. \( dx + dy = 0 \)  
402. \( ydx + xdy = 0 \)

403. \( (x + y)y' = y \)  
404. \( xy' = 2x - y \)

405. \( xy' - x^2 \sin x = y \)  
406. \( y' - 2y + 2x\sqrt{y} = 0 \)

407. \( y'' - 6y' + 25y = 0 \)  
408. \( y'' - 4y' + 4y = 4 \)

409. \( y'' - 2y' = 4x - 8 \)  
410. \( 5y'' - 6y' + y = -41e^{\frac{1}{3}x} \cos x \)

Snižavanjem reda riješi diferencijalne jednadžbe

411. \( y''' - 3y'' + 2y' = 0 \)  
412. \( y''' + y' = 1 \)

413. \( y'''' - y'' = 0 \)  
414. \( y'''' - 3y''' = 0 \)
R. RJEŠENJA

1. \( F_1(x) = \frac{1}{3}x^3 \), \( F_2(x) = (x)\frac{1}{3}x^3 + 1 \), \( F_3(x) = \frac{1}{3}x^3 - \sqrt{2} \)
   \( 2. \frac{1}{3}x^3 + C \)

3. \( F_1(x) = \sin x \), \( F_2(x) = \sin x + 2 \)
   \( 4. F(x) = \frac{1}{4}x^4 + \frac{3}{4} \)

5. \( F(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{13}{6} \)
   \( 6. F(x) = \arctan x + 1 \)
   \( 7. F(x) = e^x - 11 \)

8. \( F(x) = 2e^x - \frac{3}{2}x^2 + 5x \)
   \( 9. F(x) = -\frac{1}{2}\cos 2x \)
   \( 10. F(x) = \frac{1}{2}x - \frac{1}{4}\sin 2x \)

11. \( \text{Jest} \)
   \( 12. \text{Nije} \)
   \( 13. \text{Jest} \)
   \( 14. \frac{1}{4}x^4 + C \)
   \( 15. \frac{3}{4}\sqrt{x^4} + C \)

16. \( \frac{4}{9}\sqrt[9]{x^9} + C \)
   \( 17. -\frac{1}{3x^3} + C \)
   \( 18. \frac{2}{5}\sqrt{t^5} + C \)
   \( 19. \frac{15}{29}\sqrt{x^{29}} + C \)

20. \( 6x + C \)
   \( 21. \frac{7}{2}t^2 + C \)
   \( 22. \frac{2\sqrt{2}}{3}\sqrt{x^3} + C \)
   \( 23. x^3 - 2\cos x + C \)

24. \( \frac{1}{2}s^2 - 4\sin s + C \)
   \( 25. \frac{25}{3}x^3 - 5x^2 + x + C \)
   \( 26. \frac{8}{3}\sqrt{x^3} - 2\sqrt{x} + C \)

27. \( -\frac{1}{s} + \ln|s| + C \)
   \( 28. \frac{2}{9}\sqrt{3x}(3 - 2x) + C \)
   \( 29. \frac{1}{2}x^2 - 6x + 9\ln|x| + C \)

30. \( \frac{1}{5}x^5 + x^3 + 3x - \frac{1}{x} + C \)
   \( 31. \frac{1}{2}x^2 - 8x + \frac{32}{x} - \frac{8}{x^2} + 24\ln|x| + C \)

32. \( \frac{3x}{\ln3} + C \)
   \( 33. \frac{4x^2}{\ln4} + C \)
   \( 34. x - 2e^x + C \)

35. \( \frac{1}{3}\arctan\frac{x}{3} + C \)
   \( 36. \frac{\sqrt{2}}{4}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C \)
   \( 37. \frac{\sqrt{3}}{6}\ln\left|\frac{x+\sqrt{3}}{x-\sqrt{3}}\right| + C \)

38. \( \ln|x + \sqrt{x^2 + 4}| + C \)
   \( 39. \ln|x + \sqrt{x^2 - 4}| + C \)

40. \( \arcsin\frac{x}{2} + C \)
   \( 41. \frac{1}{2}x\sqrt{x^2 + 16} + 8\ln|x + \sqrt{x^2 + 16}| + C \)
42. \( \frac{1}{2} x \sqrt{x^2 - 16} - 8 \ln |x + \sqrt{x^2 - 16}| + C \)  
43. \( \frac{1}{2} x \sqrt{16 - x^2} + 8 \arcsin \frac{x}{4} + C \)

44. \( \frac{1}{35} (5x + 2)^7 + C \)  
45. \( -\frac{2}{9} \sqrt{(4 - 3x)^3} + C \)  
46. \( \frac{3}{8} \sqrt{(x^2 - 6)^4} + C \)

47. \( \frac{1}{2} \sin(2x - 1) + C \)  
48. \( -\frac{1}{2} \cos(x^2) + C \)  
49. \( -\frac{1}{3} \cos^3 x + C \)

50. \( \sqrt{x^2 - 9} + C \)  
51. \( -\frac{1}{2 \sin^2 t} + C \)  
52. \( \ln |\ln x| + C \)

53. \( e^x - 2 \ln(e^x + 2) + C \)  
54. \( \frac{1}{3} \ln |t^3 + 1| + C \)  
55. \( \frac{1}{3} \ln^3 x + C \)

56. \( \frac{1}{3} e^{x^3} - 3 + C \)  
57. \( -\frac{5}{12} \sqrt{1 - x^4}^3 + C \)  
58. \( -\frac{1}{5} (2 + x^2) \sqrt{(3 - x^2)^3} + C \)

59. \( \frac{1}{2} \arctan \frac{x^2}{2} + C \)  
60. \( \frac{1}{8} \ln \left| \frac{x^7}{x^4 - 1} \right| + C \)  
61. \( \ln |4 + x + \sqrt{x^2 + 8x} + C| \)

62. \( \arcsin \frac{x - 1}{2} + C \)  
63. \( \frac{1}{2} (1 + x) \sqrt{2x^2 + 2x - 1} - \ln |1 + x + \sqrt{x^2 + 2x - 1}| + C \)

64. \( \frac{1}{2} (x - 1) \sqrt{2x^2 - x^2} + \frac{1}{2} \arcsin \sin(x - 1) + C \)  
65. \( x(\ln x - 1) + C \)

66. \( (x - 1)e^x + C \)  
67. \( \sin x - x \cos x + C \)

68. \( x^2 \sin x + 2x \cos x - 2 \sin x + C \)  
69. \( \left( \frac{x^2}{\ln 2} - \frac{2x}{\ln^2 2} + \frac{2}{\ln^3 2} \right) 2^x + C \)

70. \( \frac{1}{5} x^5 \left( \log x - \frac{1}{5 \ln 10} \right) + C \)  
71. \( \frac{3}{4} x^3 (2 \ln x - 3) + C \)

72. \( -(x^2 + 3x + 3) e^{-x} + C \)  
73. \( x \tan x + \ln|\cos x| + C \)

74. \( t \arctan -\frac{1}{2} \ln(t^2 + 1) + C \)  
75. \( \frac{2x^2 - 1}{4} \arcsin x + \frac{x}{4} \sqrt{1 - x^2} + C \)

76. \( t \left( \log^2 t - \frac{2}{\ln^3 \log 3} t + \frac{2}{\ln^2 3} \right) + C \)  
77. \( \frac{1}{2} e^x (\sin x - \cos x) + C \)

78. \( \frac{1}{\ln^3 + 1} 3^x (\sin x + \ln 3 \cos x) + C \)  
79. \( \frac{1}{2} e^{-x} (\sin x - \cos x) + C \)

80. \( 3 \ln |x| + C \)  
81. \( \frac{1}{4x^4} + C \)  
82. \( \frac{5}{2} \ln |2x - 3| + C \)  
83. \( -\frac{2}{9(1 - 3x)^3} + C \)

84. \( \frac{1}{\sqrt{2}} \arctan \frac{x - 1}{\sqrt{2}} + C \)  
85. \( \frac{1}{2} (x^2 + 4x + 5) - 2 \arctan(x + 2) + C \)
86. \(- \ln(x^2 + 3) + \frac{4}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + C\)  
87. \(- \frac{1}{2} \ln(x^2 - 2x + 5) - \frac{3}{2} \arctan \frac{x - 1}{2} + C\)  

88. \(- \frac{1}{2(x^2+1)} + C\)  
89. \(\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan x + C\)  

90. \(5 \ln|x + 5| + \ln|x - 1| + C\)  
91. \(2 \ln|x + 3| - \ln|x - 1| + C\)  

92. \(\ln|x + 4| + 2 \ln|x - 3| - \frac{4}{x-3} + C\)  
93. \(\frac{x - 1}{(x-2)^2} + C\)  

94. \(\ln|x + 1| - \arctan \frac{x}{3} + C\)  
95. \(\ln \left| \frac{x+1}{x} \right| - \frac{1}{x} + C\)  

96. \(\frac{1}{2} \ln|x| - 2 \ln|x + 2| + \frac{3}{2} \ln|x - 2| + C\)  
97. \(- \frac{3}{4} \ln|x| + \frac{1}{2} \ln|x - 2| + \frac{1}{4} \ln|x + 4| + C\)  

98. \(\frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \arctan x + C\)  
99. \(\frac{1}{2} \ln(x^2 + 1) + \frac{1}{2(x^2+1)} + C\)  

100. \(\frac{1}{4} x^4 - x^3 - \ln|x + 3| + C\)  
101. \(\frac{2}{3} x^3 + 3x + 3 \ln \left| \frac{x^2}{x+2} \right| + C\)  

102. \(\frac{1}{2} x^2 + \ln|x^2 + x - 2| + C\)  
103. \(x - 4 \arctan \frac{x}{4} + C\)  

104. \(x^2 - 6x - 3 \ln|x - 1| + 12 \ln|x + 4| + C\)  
105. \(x + \ln(x^4 - x^3 - x + 1) + C\)  
106. \(\frac{1}{2} x^2 - 4 \ln|x| + 2 \arctan \frac{x}{2} + C\)  

107. \(x + 2 \ln|x| - \arctan(x-1) + C\)  
108. \(\frac{1}{3} x^3 + \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \arctan x + C\)  
109. \(\frac{1}{2} x^2 - \frac{1}{2(x^2+1)} - \ln(x^2 + 1) + C\)  

110. \(2\sqrt{x} - 2 \ln(\sqrt{x} + 1) + C\)  
111. \(2\sqrt{x - 1} - 2 \arctan\sqrt{x - 1} + C\)  

112. \(\frac{3}{4} (x - 3)^3 + \sqrt[3]{x + 1} + C\)  
113. \(6 \sqrt[3]{x + 3} \ln \left| \frac{\sqrt[3]{x-1}}{\sqrt[3]{x+1}} + C\right| + C\)  
114. \(- \frac{1}{3} \sqrt[3]{\frac{x+2}{x}}^3 + C\)  

115. \(-2 \sqrt[3]{\frac{x+2}{x}} + \ln \left| \frac{\sqrt[3]{x^2+2} + \sqrt[3]{x}}{\sqrt[3]{x^2-2} - \sqrt[3]{x}} \right| + C\)  
116. \(- \ln \left| \frac{1+\sqrt{1-x^2}}{x} \right| + C\)  

117. \(\sqrt{1 - x^2} - \ln \left| \frac{1+\sqrt{1-x^2}}{x} \right| + C\)  
118. \(\frac{1}{4} \sin 2x - \frac{1}{12} \sin 6x + C\)
119. $\frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x + C$

120. $\frac{1}{2} \sin x + \frac{1}{14} \sin 7x + C$

121. $\frac{1}{2} x + \frac{1}{20} \sin 10x + C$

122. $-\frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x + \frac{1}{24} \cos 6x + C$

123. $\frac{1}{2} \sin x - \frac{1}{12} \sin 3x - \frac{1}{20} \sin 5x + C$

124. $\frac{1}{\cos x} + C$

125. $-\frac{1}{2 \sin^2 x} + C$

126. $\frac{1}{(\cos x + 3)^3} + C$

127. $\frac{1}{2} \arctan \left( \frac{1}{2} \sin x \right) + C$

128. $\frac{1}{3} \tan^3 x + C$

129. $\cot^3 x - \cot x + C$

130. $\ln |\tan x| + C$

131. $\frac{1}{2} \arctan(2 \tan x) + C$

132. $-\frac{2}{\tan^2 x + 1} + C$

133. $\ln \left| \frac{\tan \frac{x+1}{2}}{\tan \frac{x}{2} - 1} \right| = \ln \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right| + C$

134. $x - \tan \frac{x}{2} + C$

135. $\frac{1}{5} \ln \left| \frac{2 \tan \frac{x-1}{2}}{\tan \frac{x}{2} + 2} \right| + C$

136. $\frac{2}{3} \sqrt{x^3} + C$

137. $\frac{2x+2}{3} - \frac{x-3}{ln2-ln3} + C$

138. $\frac{3}{2} \sqrt{(x-3)^2} + C$

139. $\frac{4}{9} \sqrt{(x^3 - 8)^3} + C$

140. $\frac{1}{6} \ln \left| \frac{x+5}{x-1} \right| + C$

141. $\ln(x^2 - 2x + 2) + \arctan(x - 1) + C$

142. $-4 \ln|x| + 2 \ln(x^2 + 1) + 3 \arctan x + C$

143. $x + \ln \left| 1 - \frac{1}{x^2} \right| + C$

144. $(x^3 + x) \arctan x - \frac{1}{2} x^2 + C$

145. $x \arcsin 2x + \frac{1}{2} \sqrt{1 - 4x^2} + C$

146. $\frac{1}{2} x^2 \log 4x - \frac{1}{4 \ln 10} x^2 + C$

147. $\frac{1}{\ln^2 2 + 1} 2^x (\ln 2 \sin x - \cos x) + C$

148. $4 \sqrt{x^3} + 2 \sqrt{x} + C$

149. $\sqrt{x^2 - 1} - \ln|x + \sqrt{x^2 - 1}| + C$

150. $\frac{1 - \cos x}{\sin x} + C$

151. $\frac{1 + \sin 2x}{2 \cos^2 x} + C$
152. $\frac{15}{4}$  153. $\frac{5}{12}$  154. 1  155. $-1$  156. $\frac{\pi}{3}$
157. $\frac{8}{9 \ln 3}$  158. $-\frac{24}{5}$  159. $\frac{3}{2}$  160. $\frac{116}{5}$  161. $-43$
162. 25  163. $-\frac{49}{30}$  164. $\frac{26}{3}$  165. $\frac{15}{4}$  166. $-\frac{\sqrt{2}}{12}$
167. $\frac{3}{8}$  168. $3 \ln 5 - 4$  169. $\frac{1}{2}$  170. $\frac{16}{3} - 2 \ln 3$

171. $-\frac{2}{3}$  172. $\frac{\pi}{2}$  173. $-2$  174. $\frac{e^2 + 1}{4}$
175. $\frac{e^2 - 5}{e}$  176. $\frac{9 - \ln 10}{10 \ln 10}$  177. $\frac{\pi - 2}{4}$  178. $\frac{8 \ln 2 - 4}{\ln 2}$
179. $\frac{\pi}{4}$  180. $\frac{5 - 8 \ln 2}{4}$  181. $\frac{\pi - 2 \ln 2}{12}$  182. $-\frac{e^x + 1}{2}$
183. 0  184. $-\frac{3}{2}$  185. $\frac{146}{15}$  186. 0
187. $\pi$  188. 0  189. $\frac{16}{3}$  190. $\frac{128}{3}$

191. $f(x) = \frac{2}{3} \sqrt{x^3}, f(4) = \frac{16}{3}$  192. $f(x) = \frac{1}{3} x^3 + x^2 - \frac{2}{3}, f(0) = -\frac{2}{3}$
193. $f(x) = \ln 2 - \ln |x|, f(-e) = \ln 2 - 1$
194. $f(x) = 4 + x - \frac{3}{4} \sqrt{x^4}, f(1) = \frac{17}{4}$
195. $f(x) = \frac{3x^3 - 1}{2x}, f(2) = \frac{23}{4}$
196. $f(x) = \ln \left| \frac{x + 1}{x + 2} \right| + 1, f(3) = \ln \frac{4}{5} + 1$

197. 9  198. 4  199. 24  200. 1  201. 2
202. $\frac{9}{2}$  203. $\frac{38}{3}$  204. 1  205. 16  206. 24
207. \[ P = \int_{-1}^{2} (2 + y - y^2) \, dy = \frac{9}{2} \]  
208. \[ P = \int_{0}^{1} (y-1)^2 \, dy = \frac{1}{3} \]  
209. \[ P = 2 \int_{1}^{2} (-x^2 + 3x - 2) \, dx = \frac{1}{3} \]  
210. \[ P = 4 \int_{0}^{2} (4 - x^2) \, dx = \frac{64}{3} \]  
211. \[ \frac{64}{3} \]  
212. \[ \frac{16}{3} \]  
213. \[ \frac{3}{4} \]  
214. \[ P = 2 \int_{0}^{2} (4x - x^3) \, dx = 8 \]  
215. \[ P = \int_{0}^{1} (x^3 + \sqrt{x}) \, dx + \int_{1}^{4} (2 - x + \sqrt{x}) \, dx = \frac{49}{12} \]  
216. \[ \frac{7}{3} \]  
217. \[ \frac{4}{3} \]  
218. \[ \frac{1}{3} \]  
219. \[ \frac{1}{12} \]  
220. \[ \frac{1}{2} P = \int_{-3}^{2} \sqrt{x+3} \, dx - \int_{0}^{2} \sqrt{4x} \, dx = \int_{0}^{2} \left( 3 - \frac{3}{4} y^2 \right) \, dy = 4, \ P = 8 \]  
221. \[ P = \int_{-2}^{2} \sqrt{x+2} \, dx - \int_{0}^{2} \sqrt{2x} \, dx = \int_{0}^{2} \left( 2 - \frac{1}{2} y^2 \right) \, dy = \frac{8}{3} \]  
222. \[ P = \int_{0}^{4} \sqrt{x} \, dx + \int_{4}^{6} (6-x) \, dx = \int_{0}^{2} (6 - y - y^2) \, dy = \frac{22}{3} \]  
223. \[ P = \int_{\frac{1}{2}}^{\frac{3}{2}} \left( \frac{10}{3} - \frac{2}{3} x - \frac{x+1}{x} \right) \, dx = \int_{\frac{3}{4}}^{\frac{5}{3}} \left( 5 - \frac{3}{2} y - \frac{1}{y-1} \right) \, dy = \frac{35}{12} - \ln 6 \]  
224. \[ P = \int_{0}^{\frac{\pi}{4}} \left( \frac{\pi}{2} - \arctg x \right) \, dx = \int_{0}^{\frac{\pi}{4}} tgy \, dy + \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} dy = \frac{\ln 4 + \pi}{4} \]  
225. \[ P = \int_{0}^{e} dx - \int_{1}^{e} \ln x \, dx = \int_{0}^{1} e^y \, dy = e - 1 \]  
226. \[ \frac{32\pi}{5} \]  
227. \[ \frac{625\pi}{6} \]  
228. \[ \frac{256\pi}{5} \]  
229. \[ \frac{49\pi}{30} \]  
230. \[ \frac{162}{5} \]  
231. \[ \frac{\pi}{2} \]  
232. \[ V = \pi \int_{0}^{1} x \, dx + \pi \int_{1}^{4} \left( \frac{4-x}{3} \right)^{2} \, dx = \frac{3\pi}{2} \]  
233. \[ V = \pi \int_{\frac{1}{8}}^{\frac{1}{2}} x^3 \, dx + \pi \int_{\frac{1}{8}}^{\frac{1}{1}} x^{-2} \, dx - \pi \int_{\frac{1}{8}}^{\frac{1}{4}} dx = \frac{49\pi}{80} \]
234. \[ V = \pi \int_{1}^{\frac{3}{2}} (2x+1)\,dx - \pi \int_{1}^{4} (x-1)^2\,dx = \frac{45\pi}{4} \]

235. \[ V = \pi \int_{-1}^{3} (-x^2+2x+6)^2\,dx - 2\pi \int_{-1}^{0} (x^2-2x)^2\,dx = \frac{1936\pi}{15} \]

236. \[ \frac{16\pi}{15} \]

237. \[ \frac{\pi}{2} \]

238. \[ 8\pi \]

239. \[ \frac{3\pi}{10} \]

240. \[ \frac{34\pi}{5} \]

241. \[ V = 8\pi \int_{0}^{\frac{1}{2}} y\,dy + \pi \int_{\frac{1}{2}}^{\frac{1}{3}} \frac{1}{y^2}\,dy - \pi \int_{\frac{1}{3}}^{1} \sqrt{y^2}\,dy = \frac{7\pi}{5} \]

242. \[ \frac{38}{3} \]

243. \[ \frac{19}{27} \]

244. \[ \ln (1+\sqrt{2})+\sqrt{2} \]

245. \[ \frac{1}{2} \ln (2+\sqrt{5})+\sqrt{5} \]

246. \[ \ln 3 - \frac{1}{2} \]

247. \[ \ln \frac{9}{8}+4 \]

248. \[ l = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1+\cos^2 y} \,dy = \frac{1}{2} \ln 3 \]

249. \[ l = \int_{\ln \frac{\sqrt{3}}{2}}^{\frac{\ln 2}{2}} \sqrt{\frac{1}{1-e^{2x}}}\,dx = \frac{1}{2} \ln \frac{3}{2} \]

250. \[ \frac{7\pi}{9} \]

251. \[ \frac{56\pi}{3} \]

252. \[ \frac{6\pi}{5} \]

253. \[ P = 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2}{\sin x} \sqrt{1+\frac{4\cos^2 x}{\sin^4 x}}\,dx = 4\sqrt{2}\pi \]

254. \[ 2\pi \]

255. \[ \frac{13\pi}{3} \]

256. \[ P = \frac{\pi}{3} \int_{0}^{\pi} y \sqrt{y^2+4y}\,dy = \frac{4\pi \ln (2+\sqrt{3})}{3} \]

257. \[ 0,165 \]

258. \[ -13,811 \]

259. \[ 3,093 \]

260. \[ 0,342 \]

261. \[ 8,018 \]

262. \[ 223,054 \]

263. \[ 1,585 \]

264. \[ -16,083 \]

265. \[ -0,439 \]

266. \[ 0,195 \]

267. \[ 2,063 \]

268. \[ 268,383 \]

269. \[ 21,998 \]

270. \[ 4,554 \]

271. \[ 0,334 \]

272. \[ -0,732 \]

273. \[ 0,000 \]

274. \[ -87,786 \]

275. \[ 0,635 \]

276. \[ -4,022 \]
277. $\frac{64}{5}$  278. $\frac{1793}{12}$  279. $-\frac{1}{3}$  280. 0

281. $\int_{-3}^{0} (-x)\,dx + \int_{0}^{2} x\,dx = \frac{13}{2}$  282. $\frac{5}{2}$  283. $\frac{1}{2}$  284. $\frac{1}{2}$

285. $\int_{-3}^{0} |x + 2|\,dx = \int_{-3}^{2} (-x - 2)\,dx + \int_{-2}^{0} (x + 2)\,dx = \frac{5}{2}$

286. 5  287. 2  288. 0  289. $e^3 + e - 2$  290. 1

291. $P = 4\int_{0}^{a} \sqrt{a^2 - x^2}\,dx = \pi a^2$  $l = 4a\int_{0}^{a} \frac{1}{\sqrt{a^2 - x^2}}\,dx = 2\pi a$

292. $V = \pi \frac{a^2}{v^2} \int_{0}^{v} x^2\,dx = \frac{\pi}{3} a^2 v$  $P = \pi a^2 + 2\pi a \frac{\sqrt{a^2 + v^2}}{v^2} \int_{0}^{v} x\,dx = \pi a \left(a + \sqrt{a^2 + v^2}\right)$

293. $V = 2\pi \int_{0}^{a} \left(a^2 - x^2\right)\,dx = \frac{4\pi}{3} a^3$  $P = 4\pi a \int_{0}^{a} \,dx = 4\pi a^2$

294. Promatraj vrtnju kružnice $x^2 + (y-b)^2 = a^2$ oko osi $x$

$V = 8\pi b \int_{0}^{a} \sqrt{a^2 - x^2}\,dx = 2\pi a^2 b$  $P = 8\pi ab \int_{0}^{a} \frac{1}{\sqrt{a^2 - x^2}}\,dx = 4\pi^2 ab$
295. Jesu
296. Nije
297. Jesu
298. Nisu
299. Jesu
300. Nije
301. Jesu
302. Jesu
303. Jesu
304. Jesu
305. Nije
306. Jesu
307. \( y = x^2 - \sin x + C \)
308. \( y = x - e^x + C \)
309. \( y = \ln|Cx| \)
310. \( y = \ln^2 x + \ln x + C \)
311. \( y = x^3 + x^2 + C_1x + C_2 \)
312. \( y = \frac{1}{6} x^3 + \ln|x| + C_1x + C_2 \)
313. \( y = e^x + C_1x - \ln|C_2x| \)
314. \( y = \cos x + x \sin x + C_1x + C_2 \)
315. \( y = C_1x^2 + C_2x + C_3 \)
316. \( y = \frac{1}{2} x^3 - \sin x + C_1x^2 + C_2x + C_3 \)
317. \( y = -xe^x + C_1x^2 + C_2x + C_3 \)
318. \( y = \ln^2 x + 3 \ln x + C_1x^2 + C_2x + C_3 \)
319. \( y = x^3 + x^2 - \ln|x| + 1 \)
320. \( y = \sin x + \cos x - 3 \)
321. \( y = e^{5x} - 5x^2 + 4 \)
322. \( y = x^4 - x^2 - 11x + 12 \)
323. \( y = \frac{x-1}{x^3} - \frac{1}{4} x + \frac{13}{16} \)
324. \( y = \tan x + \cot x + x - 2 \)
325. \( y = e^x - x^2 \)
326. \( y = x^3 + \frac{1}{x^3} - 6x^2 + 20x - 13 \)
327. \( 8\sqrt{x^3} - 3y^4 = C \)
328. \( x = \sqrt{y^2 - 1} + C \)
329. \( y = \frac{1}{2} \ln^2 x + C \)
330. \( \frac{x^2}{2} + \frac{y^3}{3} = C \)
331. \( x^2 - y^2 - 2(\cos x + \sin y) = C \)
332. \((x-1)e^x - ye^y = C\)
333. \( \frac{1}{x} - \frac{1}{y} = C \)
334. \( y = Ce^{x^2} \)
335. \( x^2 + y^2 = C \)
336. \( \sqrt{x^3} - 3\sqrt{y} = C \)
337. \( x = \ln(y^2 + 3) + C \)
338. \( e^{-x} + e^y = C \)
339. \( y = -x \ln|Cx| \)
340. \( x^2 = 2y^2 \ln|Cx| \)
341. \( x + y \ln|Cy| = 0 \)
342. \( \ln|Cx| = \frac{1}{2} \ln^2 \frac{x}{y} \)
343. \( y = x \ln|Cx| \)
344. \( y = -\frac{x}{\ln|Cx|} \)
345. \( y^2 = 2x^2 \ln|Cx| \)
346. \( \ln|Cx| + \cos \frac{y}{x} = 0 \)
347. \( x = y \ln|Cy| \)
348. \( x^2 + 2y^2 \ln|Cx| = 0 \)
349. \( \ln|Cx| = \tan \frac{y}{x} \)
350. \( y = 2x \arctan Cx \)
351. \( y = Ce^{-3x} \)
352. \( y = Cx \)
353. \( y = Ce^{\cos x} \)
354. \( y = Cx^x \)
355. \( y = (x^2 + C)e^x \)
356. \( y = Ce^{-2x} + 2x^2 - 2x + 1 \)
357. \( y = x^2 + x + \frac{C}{x} \)
358. \( y = \sin x + \frac{\cos x}{x} + \frac{C}{x} \)
359. \( y = (x + C)e^{x^2} \)
360. \( y = Ce^{\cos x} + 1 \)
361. \( y = x^4 + \frac{C}{x} \)
362. \( y = x(\ln^2 x + C) \)
363. \( y = (x + C)e^{\frac{1}{x}} \)
364. \( y = Ce^{\log x} - 1 \)
365.  \( y = 4e^x + 1 \)  
366.  \( y = \frac{1}{3}x^2 + \frac{2}{3}x \)  
367.  \( y = e^{\sin x} \)  
368.  \( y = -1 \)  
369.  \( y = -\frac{e^x}{e^x + C} \)  
370.  \( y = x(x + C)^2 \)  
371.  \( y = \pm e^{2x}\sqrt{x + C} \)  
372.  \( y = \frac{3}{5}x^2 + \frac{C}{x^3} \)  
373.  \( y = \pm \frac{e^{-x^2}}{\sqrt{e^{-2x^2} + C}} \)  
374.  \( y = e^x(e^{\frac{1}{3}x} + C)^3 \)  
375.  \( y = C_1 + C_2e^{2x} - e^x \)  
376.  \( y = C_1 + C_2x^2 + x^3 \)  
377.  \( y = \ln^3 x + C_1 \ln x + C_2 \)  
378.  \( y = \cos^2 x + C_1 \cos x + C_2 \)  
379.  \( y = C_1e^{2x} + C_2e^{-3x} \)  
380.  \( y = C_1e^{4x} + C_2 \)  
381.  \( y = (C_1x + C_2)e^{2x} + 1 \)  
382.  \( y = (C_1x + C_2)e^{\frac{1}{2}x} \)  
383.  \( y = C_1 \sin 5x + C_2 \cos 5x \)  
384.  \( y = (C_1 \sin 3x + C_2 \cos 3x)e^{-2x} \)  
385.  \( y = (x^2 + C_1x + C_2)e^x \)  
386.  \( y = C_1 \sin x + C_2 \cos x + x \)  
387.  \( y = C_1 + (C_2 - \frac{1}{2}x)e^{-2x} \)  
388.  \( y = C_1e^x + (2x^2 - 2x + C_2)e^{3x} \)  
389.  \( y = C_1 \sin x + (C_2 - \frac{1}{2}x)\cos x \)  
390.  \( y = C_1e^x + C_2e^{2x} + \cos x \)  
391.  \( y = (C_1 - x)\sin 2x + (C_2 + x)\cos 2x \)  
392.  \( y = C_1 + C_2e^x - \frac{1}{2}(\sin x + \cos x)e^x \)  
393.  \( y = (\frac{1}{4}x^2 - \frac{1}{2}x + C_1)e^x + C_2e^{-x} \)  
394.  \( y = C_1 + C_2e^{-x} + (e^{-x} + 1)\ln(e^x + 1) \)
395. \( y = x^2 - 3x + 4 \)
396. \( y = e^{2x} - e^x \)
397. \( y = (\sin x + \frac{1}{3})e^{3x} \)
398. \( y = e^x + C_1x + C_2 \)
399. \( y = \frac{1}{2}\ln|x| + C_1x^2 + C_2x + C_3 \)
400. \( y = e^x + C_1x + C_2 \)
401. \( x + y = C \)
402. \( xy = C \)
403. \( x = y \ln|Cy| \)
404. \( y = x - \frac{C}{x} \)
405. \( y = Cx + x \cos x \)
406. \( y = (Ce^x + x + 1)^2 \)
407. \( y = (C_1 \sin 4x + C_2 \cos 4x)e^{3x} \)
408. \( y = (C_1x + C_2)e^{2x} + 1 \)
409. \( y = C_1 + C_2e^{2x} - x^2 + 3x \)
410. \( y = C_1e^x + (C_2 + 4 \sin x + 5 \cos x)e^{\frac{1}{5}x} \)
411. \( y = C_1e^x + C_2e^{2x} + C_3 \)
412. \( y = x + C_1 \sin x + C_2 \cos x + C_3 \)
413. \( y = C_1e^x + C_2e^{-x} + C_3x + C_4 \)
414. \( y = C_1x + C_2x^2 + C_3e^{3x} + C_4 \)