

# ZI. NEODREĐENI INTEGRALI

## 1. Antidervacije

1. Pronađi tri antiderivacije funkcije  $f(x) = x^2$ .
2. Odredi sve antiderivacije funkcije  $f(x) = x^2$ .
3. Pronađi dvije antiderivacije funkcije  $f(x) = \cos x$ .
4. Pronađi antiderivaciju  $F(x)$  funkcije  $f(x) = x^3$  za koju je  $F(1) = 1$ .
5. Pronađi onu antiderivaciju  $F(x)$  funkcije  $f(x) = x^2 - x$  za koju vrijedi  $F(1) = 2$ .
6. Pronađi antiderivaciju  $F(x)$  funkcije  $f(x) = \frac{1}{1+x^2}$  koja zadovoljava uvjet  $F(0) = 1$ .
7. Ima li funkcija  $f(x) = e^x$  antiderivaciju  $F(x)$  za koju je  $F(0) = -10$ ?
8. Odredi bar jednu antiderivaciju funkcije  $f(x) = 2e^x - 3x + 5$ .
9. Odredi bar jednu antiderivaciju funkcije  $f(x) = \sin 2x$ .
10. Uz pomoć jednog trigonometrijskog identiteta pronajdi antiderivaciju funkcije  $f(x) = \sin^2 x$ .
11. Je li funkcija  $F(x) = \sqrt{x^2 - 4x + 1}$  antiderivacija funkcije  $f(x) = \frac{x-2}{\sqrt{x^2-4x+1}}$ ?
12. Je li funkcija  $F(x) = x^3 - 2x^2 - 3$  antiderivacija funkcije  $f(x) = 3x^2 - 4x - 3$ ?
13. Je li funkcija  $F(x) = 3 \sin x - \sin^3 x$  antiderivacija funkcije  $f(x) = 3 \cos^3 x$ ?

## 2. Integriranje pomoću tablice i osnovnih pravila

Služeći se tablicom i osnovnim pravilima pronađi neodređene integrale

14.  $\int x^3 dx$

15.  $\int \sqrt[3]{x} dx$

16.  $\int \sqrt[4]{x^5} dx$

17.  $\int \frac{1}{x^4} dx$

18.  $\int \frac{t^2}{\sqrt{t}} dt$

19.  $\int \frac{x^3 \sqrt{x}}{\sqrt[5]{x^2}} dx$

20.  $\int 6 dx$

21.  $\int 7t dt$

22.  $\int \sqrt{2x} dx$

23.  $\int (3x^2 + 2 \sin x) dx$

24.  $\int (s - 4 \cos s) ds$

25.  $\int (5x - 1)^2 dx$

26.  $\int \frac{4x-1}{\sqrt{x}} dx$

27.  $\int \frac{s+s^2}{s^3} ds$

28.  $\int \frac{1-2x}{\sqrt{3x}} dx$

29.  $\int \frac{(x-3)^2}{x} dx$

30.  $\int \frac{(x^2+1)^3}{x^2} dx$

31.  $\int \frac{(2-x)^4}{x^3} dx$

32.  $\int 3^x dx$

33.  $\int 4^{x+2} dx$

34.  $\int (1 - 2e^x) dx$

35.  $\int \frac{1}{x^2+9} dx$

36.  $\int \frac{1}{x^2-2} dx$

37.  $\int \frac{1}{3-x^2} dx$

38.  $\int \frac{1}{\sqrt{x^2+4}} dx$

39.  $\int \frac{1}{\sqrt{x^2-4}} dx$

40.  $\int \frac{1}{\sqrt{4-x^2}} dx$

41.  $\int \sqrt{x^2 + 16} dx$

42.  $\int \sqrt{x^2 - 16} dx$

43.  $\int \sqrt{16 - x^2} dx$

### 3. Metoda zamjene

Pogodnim zamjenama odredi integrale

- |  |  |                                  |
|--|--|----------------------------------|
| 44. $\int (5x + 2)^6 dx$               | 45. $\int \sqrt{4 - 3x} dx$                | 46. $\int x^3 \sqrt{x^2 - 6} dx$ |
| 47. $\int \cos(2x - 1) dx$             | 48. $\int x \sin(x^2) dx$                  | 49. $\int \sin x \cos^2 x dx$    |
| 50. $\int \frac{x}{\sqrt{x^2 - 9}} dx$ | 51. $\int \frac{\cos t}{\sin^3 t} dt$      | 52. $\int \frac{1}{x \ln x} dx$  |
| 53. $\int \frac{e^{2x}}{e^x + 2} dx$   | 54. $\int \frac{t^2}{t^3 + 1} dt$          | 55. $\int \frac{\ln^2 x}{x} dx$  |
| 56. $\int x^2 e^{x^3 - 3} dx$          | 57. $\int x^3 (1 - x^4)^{-\frac{2}{5}} dx$ | 58. $\int x^3 \sqrt{3 - x^2} dx$ |

Riješi integrale tako da kvadratni izraz prvo predočiš kao zbroj ili razliku kvadrata, a zatim uvedeš zamjenu

- |   |   |
|---|---|
| 59. $\int \frac{1}{x^2 - 4x + 8} dx$    | 60. $\int \frac{1}{7 - 6x - x^2} dx$        |
| 61. $\int \frac{1}{\sqrt{x^2 + 8x}} dx$ | 62. $\int \frac{1}{\sqrt{3 + 2x - x^2}} dx$ |
| 63. $\int \sqrt{x^2 + 2x - 1} dx$       | 64. $\int \sqrt{2x - x^2} dx$               |

### 4. Metoda djelomične integracije

Djelomičnim integriranjem odredi integrale

- |   |                                   |                                  |
|---|-----------------------------------|----------------------------------|
| 65. $\int \ln x dx$                     | 66. $\int x e^x dx$               | 67. $\int x \sin x dx$           |
| 68. $\int x^2 \cos x dx$                | 69. $\int x^2 2^x dx$             | 70. $\int x^4 \log x dx$         |
| 71. $\int \frac{\ln x}{\sqrt[3]{x}} dx$ | 72. $\int \frac{x^2 + x}{e^x} dx$ | 73. $\int \frac{x}{\cos^2 x} dx$ |
| 74. $\int \arctan t dt$                 | 75. $\int x \arcsin x dx$         | 76. $\int \log_3^2 t dt$         |

Dvostrukom primjenom formule za djelomičnu integraciju zadani integral svedu na integralnu jednadžbu, a potom ju riješi

- |                          |                          |                             |
|--------------------------|--------------------------|-----------------------------|
| 77. $\int e^x \sin x dx$ | 78. $\int 3^x \cos x dx$ | 79. $\int e^{-x} \cos x dx$ |
|--------------------------|--------------------------|-----------------------------|

## 5. Integriranje racionalnih funkcija

Odredi integrale djelomičnih razlomaka

80.  $\int \frac{3}{x} dx$

81.  $\int \frac{-1}{x^5} dx$

82.  $\int \frac{5}{2x-3} dx$

83.  $\int \frac{-2}{(1-3x)^4} dx$

84.  $\int \frac{1}{x^2-2x+3} dx$

85.  $\int \frac{x}{x^2+4x+5} dx$

86.  $\int \frac{-2x+4}{x^2+3} dx$

87.  $\int \frac{x+2}{-x^2+2x-5} dx$

88.  $\int \frac{x}{(x^2+1)^2} dx$

89.  $\int \frac{1}{(x^2+1)^2} dx$

Odredi integrale pravih racionalnih funkcija

90.  $\int \frac{6x}{x^2+4x-5} dx$

91.  $\int \frac{x-5}{x^2+2x-3} dx$

92.  $\int \frac{3x^2+1}{(x+4)(x-3)^2} dx$

93.  $\int \frac{-x}{(x-2)^3} dx$

94.  $\int \frac{x^2-3x+6}{(x+1)(x^2+9)} dx$

95.  $\int \frac{1}{x^3+x^2} dx$

96.  $\int \frac{7x-2}{x^3-4x} dx$

97.  $\int \frac{6}{x^3+2x^2-8x} dx$

98.  $\int \frac{1}{x^4-1} dx$

99.  $\int \frac{x^3}{(x^2+1)^2} dx$

Odredi integrale racionalnih funkcija

100.  $\int \frac{x^4-9x^2-1}{x+3} dx$

101.  $\int \frac{2x^4-5x^2}{x^2-4} dx$

102.  $\int \frac{x^3+x^2+1}{x^2+x-2} dx$

103.  $\int \frac{x^2}{x^2+16} dx$

104.  $\int \frac{2x^3-17x}{x^2+3x-4} dx$

105.  $\int \frac{x^3+4x^2+x}{x^3-1} dx$

106.  $\int \frac{x^4-x^3-16}{x^3+4x} dx$

107.  $\int \frac{x^3-3x+4}{x^3-2x^2+2x} dx$

108.  $\int \frac{x^6}{x^4-1} dx$

109.  $\int \frac{x^5}{(x^2+1)^2} dx$

## 6. Integriranje funkcija s korijenom

Pogodnim zamjenama zadane integrale svedi na integrale racionalnih funkcija i riješi ih

110.  $\int \frac{1}{\sqrt{x+1}} dx$

111.  $\int \frac{\sqrt{x-1}}{x} dx$

112.  $\int \frac{x}{\sqrt[3]{(x+1)^2}} dx$

113.  $\int \frac{\sqrt{x}}{x(\sqrt[3]{x}-1)} dx$

114.  $\int \frac{1}{x^2} \sqrt{\frac{x+2}{x}} dx$

115.  $\int \frac{1}{x} \sqrt{\frac{x+2}{x}} dx$

116.  $\int \frac{1}{x\sqrt{1-x^2}} dx$

117.  $\int \frac{\sqrt{1-x^2}}{x} dx$

## 7. Integriranje trigonometrijskih funkcija

Uz pomoć formula koje umnožak sinusa i kosinusa pretvaraju u zbroj ili razliku riješi integrale

118.  $\int \sin 2x \sin 4x dx$

119.  $\int \sin x \cos 3x dx$

120.  $\int \cos 3x \cos 4x dx$

121.  $\int \cos^2 5x dx$

122.  $\int \sin x \sin 2x \sin 3x dx$

123.  $\int \sin^2 2x \cos x dx$

Uz pomoć neke od zamjena  $t = \sin x$ ,  $t = \cos x$ ,  $t = \operatorname{tg} x$  ili  $t = \operatorname{tg} \frac{x}{2}$  riješi integrale

124.  $\int \frac{\sin x}{\cos^2 x} dx$

125.  $\int \frac{\cos x}{\sin^3 x} dx$

126.  $\int \frac{3 \sin x}{(\cos x + 3)^4} dx$

127.  $\int \frac{\cos x}{5 - \cos^2 x} dx$

128.  $\int \frac{\sin^2 x}{\cos^4 x} dx$

129.  $\int \frac{1 - 4 \cos^2 x}{\sin^4 x} dx$

130.  $\int \frac{1}{\sin x \cos x} dx$

131.  $\int \frac{1}{3 \sin^2 x + 1} dx$

132.  $\int \frac{1}{\sin x + 1} dx$

133.  $\int \frac{1}{\cos x} dx$

134.  $\int \frac{\cos x}{\cos x + 1} dx$

135.  $\int \frac{1}{3 \sin x - 4 \cos x} dx$

## 8. Različiti zadatci

Riješi integrale

$$136. \int \sqrt[3]{x\sqrt{x}} dx$$

$$138. \int \frac{1}{\sqrt[3]{x-3}} dx$$

$$140. \int \frac{1}{5-4x-x^2} dx$$

$$142. \int \frac{3x-4}{x^3+x} dx$$

$$144. \int (3x^2 + 1) \arctg x dx$$

$$146. \int x \log 4x dx$$

$$148. \int \frac{\sqrt[3]{x}+1}{\sqrt{x}} dx$$

$$150. \int \frac{1-\cos x}{\sin^2 x} dx$$

$$137. \int 2^{x+2} 3^{-x-3} dx$$

$$139. \int \frac{x^2}{\sqrt[4]{x^3-8}} dx$$

$$141. \int \frac{2x-1}{x^2-2x+2} dx$$

$$143. \int \frac{x^3-x+2}{x^3-x} dx$$

$$145. \int \arcsin 2x dx$$

$$147. \int 2^x \sin x dx$$

$$149. \int \sqrt{\frac{x-1}{x+1}} dx$$

$$151. \int \frac{\sin x + \cos x}{\cos^3 x} dx$$

## ZII. ODREĐENI INTEGRALI

### 1. Računanje određenog integrala

Služeći se tablicom, osnovnim pravilima i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

$$152. \int_{-1}^2 x^3 dx$$

$$153. \int_0^1 \sqrt[5]{x^7} dx$$

$$154. \int_1^e x^{-1} dx$$

$$155. \int_{\frac{\pi}{2}}^{\pi} \cos x dx$$

$$156. \int_0^{\sqrt{3}} \frac{1}{x^2+1} dx$$

$$157. \int_{-2}^0 3^x dx$$

$$158. \int_0^4 x(1-\sqrt{x}) dx$$

$$159. \int_4^9 (3-\sqrt{x})^2 dx$$

$$160. \int_{-1}^1 (2-\sqrt[3]{x})^3 dx$$

$$161. \int_1^{64} \left( \sqrt[4]{x} - \frac{4}{x} \right)^2 dx$$

$$162. \int_1^{16} \left( \sqrt[4]{x} - \frac{4}{x} \right)^2 dx$$

$$163. \int_1^4 \frac{(\sqrt{x}-x)^3}{x} dx$$

Služeći se metodom zamjene i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

$$164. \int_{-2}^2 \sqrt{2x+5} dx$$

$$165. \int_{-2}^1 \sqrt[3]{2-3x} dx$$

$$166. \int_{\frac{\pi}{4}}^{2\pi} \sin^2 x \cdot \cos x dx$$

$$167. \int_{-3}^{-2} \frac{1}{(x+4)^3} dx$$

$$168. \int_{-1}^3 \frac{x^2-1}{x+2} dx$$

$$169. \int_1^e \frac{\ln x}{x} dx$$

$$170. \int_0^4 \frac{x}{\sqrt{x+1}} dx$$

$$171. \int_{-4}^1 \frac{1+x}{\sqrt{5-x}} dx$$

$$172. \int_1^{27} \frac{\sqrt[6]{x}}{x(1+\sqrt[3]{x})} dx$$

Služeći se metodom djelomične integracije i Leibniz-Newtonovom formulom izračunaj vrijednost određenih integrala

$$173. \int_0^{\pi} x \cos x dx$$

$$174. \int_1^e x \ln dx$$

$$175. \int_{-1}^1 x^2 e^x dx$$

$$176. \int_1^{10} \frac{\log x}{x^2} dx$$

$$177. \int_{-1}^0 x \arctan x dx$$

$$178. \int_1^4 \frac{\log_2 x}{\sqrt{x}} dx$$

Izračunaj integrale:

$$179. \int_0^{\frac{\pi}{2}} x \sin 2x dx$$

$$180. \int_{-1}^0 x \ln(x+2) dx$$

$$181. \int_{\frac{5}{3}}^2 \arctan(3x+5) dx$$

$$182. \int_0^{\pi} e^x \cos x dx$$

$$183. \int_{-5}^5 \frac{x}{x^2+3} dx$$

$$184. \int_0^3 \frac{x^3-3x-2}{x+1} dx$$

Izračunaj integrale tako da prvo provjeriš parnost podintegralne funkcije ili njenih pribrojnika

$$185. \int_{-1}^1 (x^4 - x^2 + 5) dx$$

$$186. \int_{-3}^3 (x^3 - x \cos x) dx$$

$$187. \int_{-\pi}^{\pi} (\sin^2 x - 3 \cos x) dx$$

$$188. \int_{-5}^5 (x + \sin x - x^2 \operatorname{tg} x) dx$$

$$189. \int_{-2}^2 (x^2 \sin x - 4 \operatorname{ctg} x + \sqrt{2-x}) dx$$

$$190. \int_{-4}^4 x^2 (\sin x + \cos x)^2 dx$$

Odredi funkciju  $f(x)$  i izračunaj  $f(x_0)$ , ako je

$$191. f(x) = \int_0^x \sqrt{t} dt, \quad x_0 = 4$$

$$192. f(x) = \int_{-1}^x (t^2 + 2t) dt, \quad x_0 = 0$$

$$193. f(x) = \int_x^{-2} \frac{1}{t} dt, \quad x_0 = -e$$

$$194. f(x) = \int_x^8 (\sqrt[3]{t} - 1) dt, \quad x_0 = 1$$

$$195. f(x) = \int_x^{2x} \frac{t^3 - 1}{t^2} dt, \quad x_0 = 2$$

$$196. f(x) = \int_x^{x+1} \frac{t}{t+1} dt, \quad x_0 = 3$$



## 2. Površina ravninskog lika

Izračunaj površinu lika omeđenog krivuljama

197.  $x = 3, y = 0, y = x^2$

199.  $x = -1, x = 3, y = 0,$   
 $y = 3x^2 - 2x + 1$

201.  $y = 0, y = \sin x$  za  $0 \leq x \leq \pi$

203.  $x = 1, x = 3, y = 1 - 2x,$   
 $y = x^2 - 2x + 3$

205.  $x = 0, y = x^2, y = (x - 4)^2$

207.  $x = y + 2, x = y^2$

209.  $y = x^2 - 3x + 2,$   
 $y = -x^2 + 3x - 2$

211.  $x = y^2 - 5y + 6,$   
 $x = -y^2 + 7y - 4$

213.  $x = 0, y = 0, y = x^3 - 1$

215.  $y = 2 - x, y = -\sqrt{x}, y = x^3$

198.  $x = 2, y = 0, y = x^3$

200.  $x = 1, x = e, y = 0, y = x^{-1}$

202.  $y = -x + 3, y = x^2 - 6x + 7$

204.  $x = 1, x = e, xy = 2, xy = 3$

206.  $x = -2, x = 2, y = 3x + 4,$   
 $y = \sin x - 2$

208.  $x = 2y, x = y^2 + 1, y = 0$

210.  $y = x^2 + x - 2, y = -x^2 + x + 6$

212.  $y = x - 1, y^2 = 2x + 1$

214.  $y = 3x, y = x^3 - x$

216.  $y = |x|, y = 2 - x^2$

217.  $y = x^2 - x^3$ , tangenta u točki  $T(1, 0)$

U narednim zadatcima površinu lika omeđenog zadanim krivuljama izračunaj na dva načina:

integriranjem funkcija  $y(x)$  po  $x$

integriranjem funkcija  $x(y)$  po  $y$

218.  $x = y^2, y = x^2$

219.  $y = x^2, y = x^3$

220.  $y^2 = x + 3, y^2 = 4x$

221.  $y = 0, y = \sqrt{x+2}, y = \sqrt{2x}$

222.  $y = 0, y = -x + 6, y = \sqrt{x}$

223.  $2x + 3y = 10, xy = x + 1$

224.  $x = 0, x = 1, x = \tan y, y = \pi / 2$

225.  $x = 0, y = 0, y = 1, y = \ln x$

### 3. Obujam rotacijskog tijela

Izračunaj obujam tijela nastalog vrtnjom, oko osi  $x$ , lika omeđenog krivuljama

226.  $x = 2, y = 0, y = x^2$

227.  $y = 0, y = x^2 - 5x$

228.  $y = 4, y = x^2$

229.  $y = x, y = x^2 - x$

230.  $y = x + 4, y = x^2 + 2$

231.  $x = 0, x = 1 - y^2$

232.  $y = 0, 3y = 4 - x, y = \sqrt{x}$

233.  $xy = 1, 2y = 1, y = \sqrt[3]{x}$

234.  $y = x - 1, y^2 = 2x + 1$

235.  $y = x^2 - 2x, y = -x^2 + 2x + 6$

Izračunaj obujam tijela nastalog vrtnjom, oko osi  $y$ , lika omeđenog krivuljama

236.  $x = 0, x = 1 - y^2$

237.  $xy = 1, y = 1, y = 2$

238.  $x = 2, y = 0, y = x^2$

239.  $x = y^2, y = x^2$

240.  $y = 0, 3y = 4 - x, y = \sqrt{x}$

241.  $xy = 1, 8y = x^2, y = x^3$

#### 4. Duljina luka ravninske krivulje

Izračunaj duljinu luka krivulje

242.  $y = \frac{2}{3}\sqrt{x^3}$  za  $3 \leq x \leq 8$
243.  $x = \sqrt{y^3}$  za  $0 \leq y \leq \frac{5}{9}$
244.  $y^2 = 4x$  između točaka  $A(0,0)$  i  $B(1,2)$
245.  $y = \frac{1}{2}x^2 - x$  između točaka  $A\left(0, -\frac{1}{2}\right)$  i  $B\left(3, \frac{3}{2}\right)$
246.  $y = \ln(1-x^2)$  za  $0 \leq x \leq \frac{1}{2}$
247.  $y = 2e^{\frac{1}{2}x}$  za  $\ln 24 \leq x \leq \ln 48$
248.  $x = \ln \sin y$  za  $\frac{\pi}{3} \leq y \leq \frac{\pi}{2}$
249.  $y = \arcsin e^x$  za  $\ln \frac{\sqrt{3}}{2} \leq x \leq \ln \frac{2\sqrt{2}}{3}$

#### 5. Površina rotacijske plohe

Izračunaj površinu plohe nastale vrtnjom, oko osi  $x$ , luka krivulje

250.  $y = \frac{1}{3}x^3$  za  $0 \leq x \leq \sqrt[4]{3}$
251.  $y^2 = 4x$  između točaka  $A(3, -2\sqrt{3})$  i  $B(3, 2\sqrt{3})$
252.  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$  za  $0 \leq x \leq 1$
253.  $y = \frac{2}{\sin x}$  za  $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

Izračunaj površinu plohe nastale vrtnjom, oko osi  $y$ , luka krivulje

254.  $x = \sqrt{1-y^2}$  za  $0 \leq y \leq 1$
255.  $y = x^2$  između točaka  $A(-\sqrt{2}, 2)$  i  $B(\sqrt{2}, 2)$
256.  $x = \frac{1}{3}\sqrt{y^3}$  za  $0 \leq y \leq 2$

## 6. Numerička integracija

Trapeznom formulom, uz zadani korak  $h$ , izračunaj približnu vrijednost određenih integrala

$$257. \quad \int_2^3 \sin(x^2 + 1) dx, \quad h = 0,2$$

$$259. \quad \int_1^{1,5} \ln^2(x^3 + 10) dx, \quad h = 0,1$$

$$261. \quad \int_4^5 \frac{x^2 - 15}{\log x} dx, \quad h = 0,2$$

$$263. \quad \int_1^2 \frac{e^x}{\sin x + 2} dx, \quad h = 0,2$$

$$265. \quad \int_0^{0,7} (x - 3 \arctan x) dx, \quad h = 0,1$$

$$258. \quad \int_3^5 x^2 \cos x dx, \quad h = 0,4$$

$$260. \quad \int_{-3}^{-2} \sqrt{-x^2 - 5x - 6} dx, \quad h = 0,25$$

$$262. \quad \int_{10}^{12} \frac{x^3 + 3}{x + 1} dx, \quad h = 0,4$$

$$264. \quad \int_3^6 \frac{1 - 2^x}{x} dx, \quad h = 0,5$$

$$266. \quad \int_{-1}^0 \arcsin(x + e^x) dx, \quad h = 0,2$$

Simpsonovom formulom, uz zadani korak  $h$ , izračunaj približnu vrijednost određenih integrala

$$267. \quad \int_1^2 e^x \ln x dx, \quad h = 0,25$$

$$269. \quad \int_5^6 \frac{x^3 - 2}{x + 2} dx, \quad h = 0,25$$

$$271. \quad \int_0^{0,8} \sqrt{x - x^2} dx, \quad h = 0,1$$

$$273. \quad \int_{-1}^1 x \tan^2 x dx, \quad h = 0,5$$

$$275. \quad \int_{0,5}^{1,1} \frac{\sqrt{x + \ln x}}{\cos x} dx, \quad h = 0,1$$

$$268. \quad \int_0^{11} x^2 \log x dx, \quad h = 0,5$$

$$270. \quad \int_3^{4,2} \frac{3^x + 3}{x^2 + 2} dx, \quad h = 0,2$$

$$272. \quad \int_{-2}^0 \sqrt[3]{x + e^x} dx, \quad h = 0,5$$

$$274. \quad \int_2^3 4^x \cot x dx, \quad h = 0,25$$

$$276. \quad \int_4^5 \frac{\sin x - x}{\arctan x} dx, \quad h = 0,25$$

## 7. Različiti zadatci

Izračunaj vrijednost integrala:

$$277. \int_0^4 x\sqrt{x} dx$$

$$279. \int_{-1}^0 x\sqrt{1-x^2} dx$$

$$281. \int_{-3}^2 |x| dx$$

$$283. \int_1^2 \sqrt{x^2 - 2x + 1} dx$$

$$285. \int_{-3}^0 \sqrt{x^2 + 4x + 4} dx$$

$$287. \int_{\pi}^{2\pi} \sqrt{1 - \cos^2 x} dx$$

$$289. \int_{-1}^3 e^{|x|} dx$$

$$278. \int_1^{64} (\sqrt{x} - \sqrt[3]{x}) dx$$

$$280. \int_0^2 (x-1)\sqrt{2x-x^2} dx$$

$$282. \int_1^4 |x-3| dx$$

$$284. \int_0^1 \sqrt{x^2 - 2x + 1} dx$$

$$286. \int_{-1}^2 \sqrt{4x^2 - 4x + 1} dx$$

$$288. \int_0^{\pi} \sqrt{1 - \sin^2 x} \cdot \sin 2x dx$$

$$290. \int_1^3 |x^2 - 3x + 2| dx$$

Uz pomoć integralnog računa izvedi formule za

**291.** površinu i opseg kruga

**292.** obujam i površinu uspravnog kružnog stožca

**293.** obujam i površinu kugle

**294.** obujam i površinu torusa

## ZIII. DIFERENCIJALNE JEDNADŽBE

### 1. Provjera rješenja

Provjeri jesu li funkcije  $y$  rješenja diferencijalnih jednadžbi

$$295. \quad y' + y = 2 \cos x \quad , \quad y = \sin x + \cos x$$

$$296. \quad xy' + y = 0 \quad , \quad y = \frac{1}{x} + \frac{1}{x^2}$$

$$297. \quad y' - 2y = 2x - 1 \quad , \quad y = Ce^{2x} - x$$

$$298. \quad y'tgx - 3y + 2 = 0 \quad , \quad y = C \sin^3 x + 1$$

$$299. \quad y'' + y + 2 \sin x = 0 \quad , \quad y = x \cos x$$

$$300. \quad y'' - 2y' + y = e^x \quad , \quad y = x^2 e^x$$

$$301. \quad y'' \cos x - 2y' \sin x = 0 \quad , \quad y = \tan x$$

$$302. \quad y'' + y = x^2 + 2 \quad , \quad y = C_1 \sin x + C_2 \cos x + x^2$$

$$303. \quad 4x^2 y'' + y' = 0 \quad , \quad y = \sqrt{x^3}$$

$$304. \quad x^3 y'' + x^2 y' + xy = 3 \quad , \quad y = \frac{3}{2} \ln x$$

$$305. \quad y'' + y' + y = \cos x \quad , \quad y = \sin x$$

$$306. \quad y'' - 5y' + 6y = 0 \quad , \quad y = C_1 + C_2 e^{2x} + C_3 e^{3x}$$

## 2. Diferencijalne jednačbe koje se rješavaju neposrednim integriranjem

Neposrednim integriranjem odredi opće rješenje diferencijalnih jednačbi

307.  $y' = 2x - \cos x$

308.  $y' + e^x = 1$

309.  $xy' = 1$

310.  $xy' = 2 \ln x + 1$

311.  $y'' = 6x + 2$

312.  $y'' + x^{-2} = x$

313.  $x^2 y'' = x^2 e^x + 1$

314.  $y'' = \cos x - x \sin x$

315.  $y''' = 0$

316.  $y''' - 3 = \cos x$

317.  $y''' + (x+3)e^x = 0$

318.  $x^3 y''' = 4 \ln x$

Pronađi pojedinačno rješenje diferencijalnih jednačbi koje zadovoljava zadane uvjete

319.  $y' = 3x^2 + 2x - \frac{1}{x}$  :  $y(1) = 3$

320.  $y' = \cos x - \sin x$  :  $y(\pi) = 2y(0)$

321.  $y' = 5e^{5x} - 10x$  :  $y(0) = y'(0)$

322.  $y'' = 12x^2 - 2$  :  $y(1) = 1, y(2) = 2$

323.  $y'' = \frac{2x-6}{x^4}$  :  $y(1) = y(2), y(4) = 0$

324.  $y'' = \frac{2 \sin x}{\cos^3 x} + \frac{2 \cos x}{\sin^3 x}$  :  $y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}, y'\left(\frac{\pi}{4}\right) = 1$

325.  $y''' = e^x$  :  $y(0) = 1, y'(0) = 1, y''(0) = -1$

326.  $y''' = 6 - \frac{60}{x^6}$  :  $y(1) = 3, y'(1) = 8, y''(2) = \frac{3}{8}$

### 3. Diferencijalne jednađbe prvog reda

#### 3.1. Diferencijalna jednađba s razdvojenim promjenljivim

Riješi diferencijalne jednađbe

327.  $\sqrt{x}dx = y^3 dy$

328.  $\sqrt{y^2 - 1}dx = ydy$

329.  $\ln x dx - x dy = 0$

330.  $\frac{dx}{y^2} + \frac{dy}{x} = 0$

331.  $(x + \sin x)dx = (y + \cos y)dy$

332.  $xe^x dx (y + 1)e^y dy$

Riješi diferencijalne jednađbe tako da prvo razdvojiš diferencijale i promjenljive

333.  $x^2 y' = y^2$

334.  $y' = 2xy$

335.  $x + yy' = 0$

336.  $y' = \sqrt{xy}$

337.  $2yy' = y^2 + 3$

338.  $e^{x+y} y' = 1$



### 3.2. Homogena diferencijalna jednačba

Zamjenom  $z = \frac{x}{y}$ , a potom razdvajanjem promjenljivih  $x$  i  $z$ , riješi diferencijalne jednačbe

$$339. \quad y' = \frac{y-x}{x}$$

$$340. \quad y' = \frac{(x^2 - y^2)y}{x^3}$$

$$341. \quad y' = \frac{y}{x-y}$$

$$342. \quad y' \ln \frac{y}{x} = \frac{y}{x} (\ln \frac{x}{y} - 1)$$

Zamjenom  $z = \frac{y}{x}$ , a potom razdvajanjem promjenljivih  $x$  i  $z$ , riješi diferencijalne jednačbe

$$343. \quad y' = \frac{x+y}{x}$$

$$344. \quad y' = \frac{xy+y}{x^2}$$

$$345. \quad y' = \frac{x^2 + y^2}{xy}$$

$$346. \quad y' \sin \frac{y}{x} = \frac{y}{x} \sin \frac{x}{y} + 1$$

Riješi diferencijalne jednačbe

$$347. \quad (x+y)y' = y$$

$$348. \quad x^3 y' = (x^2 + y^2)y$$

$$349. \quad y' = \frac{y}{x} + \cos^2 \frac{y}{x}$$

$$350. \quad y' = \frac{y}{x} + \sin \frac{y}{x}$$

### 3.3. Linearna diferencijalna jednačnja

Odredi opće rješenje homogenih linearnih diferencijalnih jednačnji

$$351. \quad y' + 3y = 0$$

$$352. \quad y' - \frac{1}{x}y = 0$$

$$353. \quad y' + (\sin x)y = 0$$

$$354. \quad y' - (\ln x + 1)y = 0$$

Pronađi opće rješenje diferencijalnih jednačnji tako da prvo riješiš njihove homogene jednačnje, a zatim primijeniš metodu varijacije konstante

$$355. \quad y' - y = 2xe^x$$

$$356. \quad y' + 2y = 4x^2$$

$$357. \quad y' + \frac{1}{x}y = 3x + 2$$

$$358. \quad y' + \frac{1}{x}y = \cos x$$

$$359. \quad y' - 2xy = e^{x^2}$$

$$360. \quad y' + (\sin x)y = \sin x$$

$$361. \quad xy' + y = 5x^4$$

$$362. \quad xy' - y = 2x \ln x$$

$$363. \quad x^2y' + y = x^2e^{\frac{1}{x}}$$

$$364. \quad y' \cos^2 x - y - 1 = 0$$

Pronađi pojedinačno rješenje diferencijalnih jednačnji koje zadovoljava zadani uvjet

$$365. \quad y' - y = -1 \quad : \quad y(0) = 5$$

$$366. \quad y' + \frac{1}{x}y = x \quad : \quad y(1) = 1$$

$$367. \quad y' - (\cos x)y = 0 \quad : \quad y'(0) = 1$$

$$368. \quad y' - \frac{1}{x}y = \frac{1}{x} \quad : \quad y'(2) = 0$$

### 3.4. Bernoullieva diferencijalna jednačba

Pronađi opće rješenje diferencijalnih jednačbi tako da prvo riješiš njihove homogene jednačbe, a zatim primijeniš metodu varijacije konstante  $C$  te razdvojiš promjenljive  $x$  i  $C$

$$369. \quad y' - y = y^2$$

$$370. \quad xy' - y = 2x\sqrt{xy}$$

$$371. \quad y' - 2y = \frac{e^{4x}}{2y}$$

$$372. \quad y' + \frac{1}{x}y = \frac{x}{y^2}$$

$$373. \quad y' + 2xy - 2xy^3 = 0$$

$$374. \quad y' - y + \sqrt[3]{y^2} = 0$$

## 4. Diferencijalne jednačbe drugog reda

### 4.1. Linearna diferencijalna jednačba

Snižavanjem reda riješi linearne diferencijalne jednačbe

$$375. \quad y'' - 2y' = e^x$$

$$376. \quad xy'' - y' = 3x^2$$

$$377. \quad x^2y'' + xy' = 6 \ln x$$

$$378. \quad y'' \sin x - y' \cos x = 2 \sin^3 x$$

## 4.2. Linearna diferencijalna jednadžba s konstantnim koeficijentima

Odredi opće rješenje homogenih linearnih diferencijalnih jednadžbi

$$379. \quad y'' + y' - 6y = 0$$

$$380. \quad y'' - 4y' = 0$$

$$381. \quad \frac{1}{3}y'' - 2y' + 3y = 0$$

$$382. \quad y'' - y' + \frac{1}{4}y = 0$$

$$383. \quad y'' + 25y = 0$$

$$384. \quad y'' + 4y' + 13y = 0$$

Pronađi opće rješenje diferencijalnih jednadžbi tako da prvo riješiš njihove homogene jednadžbe, a zatim primijeniš metodu varijacije konstanti

$$385. \quad y'' - 2y' + y = 2e^x$$

$$386. \quad y'' + y = x$$

$$387. \quad y'' + 2y' = e^{-2x}$$

$$388. \quad y'' - 4y' + 3y = 8xe^{3x}$$

$$389. \quad y'' + y = \sin x$$

$$390. \quad y'' - 3y' + 2y = 3\sin x + \cos x$$

$$391. \quad y'' + 4y = -4\sin 2x - 4\cos 2x$$

$$392. \quad y'' - y' = e^x \sin x$$

$$393. \quad 2y'' - 2y = (2x - 1)e^x$$

$$394. \quad (e^x + 1)(y'' + y') = e^x$$

Pronađi pojedinačno rješenje diferencijalnih jednadžbi koje zadovoljava zadane uvjete

$$395. \quad y'' + y' = 2x - 1 \quad : \quad y(0) = 4, \quad y(1) = 2$$

$$396. \quad y'' - 2y' + y = e^{2x} \quad : \quad y(0) = 0, \quad y'(0) = 1$$

$$397. \quad y'' - 6y' + 9y = -e^{3x} \sin x \quad : \quad y'(0) = 2, \quad y'(\pi) = 0$$

$$398. \quad y'' + y = 1 \quad : \quad y'(0) = 1, \quad y''(0) = 2$$

## 5. Različiti zadatci

Riješi diferencijalne jednačbe

399.  $x^3 y''' = 1$

400.  $\ln(y'') = x$

401.  $dx + dy = 0$

402.  $ydx + xdy = 0$

403.  $(x + y)y' = y$

404.  $xy' = 2x - y$

405.  $xy' - x^2 \sin x = y$

406.  $y' - 2y + 2x\sqrt{y} = 0$

407.  $y'' - 6y' + 25y = 0$

408.  $y'' - 4y' + 4y = 4$

409.  $y'' - 2y' = 4x - 8$

410.  $5y'' - 6y' + y = -41e^{\frac{1}{5}x} \cos x$

Snižavanjem reda riješi diferencijalne jednačbe

411.  $y''' - 3y'' + 2y' = 0$

412.  $y''' + y' = 1$

413.  $y^{IV} - y'' = 0$

414.  $y^{IV} - 3y''' = 0$

## R. RJEŠENJA

1.  $F_1(x) = \frac{1}{3}x^3$ ,  $F_2 = (x)\frac{1}{3}x^3 + 1$ ,  $F_3(x) = \frac{1}{3}x^3 - \sqrt{2}$  2.  $\frac{1}{3}x^3 + C$
3.  $F_1(x) = \sin x$ ,  $F_2(x) = \sin x + 2$  4.  $F(x) = \frac{1}{4}x^4 + \frac{3}{4}$
5.  $F(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{13}{6}$  6.  $F(x) = \arctan x + 1$  7.  $F(x) = e^x - 11$
8.  $F(x) = 2e^x - \frac{3}{2}x^2 + 5x$  9.  $F(x) = -\frac{1}{2}\cos 2x$  10.  $F(x) = \frac{1}{2}x - \frac{1}{4}\sin 2x$
11. Jest 12. Nije 13. Jest 14.  $\frac{1}{4}x^4 + C$  15.  $\frac{3}{4}\sqrt[3]{x^4} + C$
16.  $\frac{4}{9}\sqrt[4]{x^9} + C$  17.  $-\frac{1}{3x^3} + C$  18.  $\frac{2}{5}\sqrt{t^5} + C$  19.  $\frac{15}{29}\sqrt[15]{x^{29}} + C$
20.  $6x + C$  21.  $\frac{7}{2}t^2 + C$  22.  $\frac{2\sqrt{2}}{3}\sqrt{x^3} + C$  23.  $x^3 - 2\cos x + C$
24.  $\frac{1}{2}s^2 - 4\sin s + C$  25.  $\frac{25}{3}x^3 - 5x^2 + x + C$  26.  $\frac{8}{3}\sqrt{x^3} - 2\sqrt{x} + C$
27.  $-\frac{1}{s} + \ln|s| + C$  28.  $\frac{2}{9}\sqrt{3x}(3 - 2x) + C$  29.  $\frac{1}{2}x^2 - 6x + 9\ln|x| + C$
30.  $\frac{1}{5}x^5 + x^3 + 3x - \frac{1}{x} + C$  31.  $\frac{1}{2}x^2 - 8x + \frac{32}{x} - \frac{8}{x^2} + 24\ln|x| + C$
32.  $\frac{3^x}{\ln 3} + C$  33.  $\frac{4^{x+2}}{\ln 4} + C$  34.  $x - 2e^x + C$
35.  $\frac{1}{3}\arctan \frac{x}{3} + C$  36.  $\frac{\sqrt{2}}{4}\ln \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| + C$  37.  $\frac{\sqrt{3}}{6}\ln \left| \frac{x+\sqrt{3}}{x-\sqrt{3}} \right| + C$
38.  $\ln|x + \sqrt{x^2 + 4}| + C$  39.  $\ln|x + \sqrt{x^2 - 4}| + C$
40.  $\arcsin \frac{x}{2} + C$  41.  $\frac{1}{2}x\sqrt{x^2 + 16} + 8\ln|x + \sqrt{x^2 + 16}| + C$

42.  $\frac{1}{2}x\sqrt{x^2-16} - 8\ln|x + \sqrt{x^2-16}| + C$       43.  $\frac{1}{2}x\sqrt{16-x^2} + 8\arcsin\frac{x}{4} + C$
44.  $\frac{1}{35}(5x+2)^7 + C$       45.  $-\frac{2}{9}\sqrt{(4-3x)^3} + C$       46.  $\frac{3}{8}\sqrt[3]{(x^2-6)^4} + C$
47.  $\frac{1}{2}\sin(2x-1) + C$       48.  $-\frac{1}{2}\cos(x^2) + C$       49.  $-\frac{1}{3}\cos^3 x + C$
50.  $\sqrt{x^2-9} + C$       51.  $-\frac{1}{2\sin^2 t} + C$       52.  $\ln|\ln x| + C$
53.  $e^x - 2\ln(e^x + 2) + C$       54.  $\frac{1}{3}\ln|t^3 + 1| + C$       55.  $\frac{1}{3}\ln^3 x + C$
56.  $\frac{1}{3}e^{x^3-3} + C$       57.  $-\frac{5}{12}\sqrt[5]{(1-x^4)^3} + C$       58.  $-\frac{1}{5}(2+x^2)\sqrt{(3-x^2)^3} + C$
59.  $\frac{1}{2}\arctan\frac{x-2}{2} + C$       60.  $\frac{1}{8}\ln\left|\frac{x+7}{x-1}\right| + C$       61.  $\ln|4+x+\sqrt{x^2+8x}| + C$
62.  $\arcsin\frac{x-1}{2} + C$       63.  $\frac{1}{2}(1+x)\sqrt{x^2+2x-1} - \ln|1+x+\sqrt{x^2+2x-1}| + C$
64.  $\frac{1}{2}(x-1)\sqrt{2x-x^2} + \frac{1}{2}\arcsin(x-1) + C$       65.  $x(\ln x - 1) + C$
66.  $(x-1)e^x + C$       67.  $\sin x - x\cos x + C$
68.  $x^2\sin x + 2x\cos x - 2\sin x + C$       69.  $\left(\frac{x^2}{\ln 2} - \frac{2x}{\ln^2 2} + \frac{2}{\ln^3 2}\right)2^x + C$
70.  $\frac{1}{5}x^5\left(\log x - \frac{1}{5\ln 10}\right) + C$       71.  $\frac{3}{4}\sqrt[3]{x^2}(2\ln x - 3) + C$
72.  $-(x^2 + 3x + 3)e^{-x} + C$       73.  $x\tan x + \ln|\cos x| + C$
74.  $t \arctan t - \frac{1}{2}\ln(t^2 + 1) + C$       75.  $\frac{2x^2-1}{4}\arcsin x + \frac{x}{4}\sqrt{1-x^2} + C$
76.  $t\left(\log_3^2 t - \frac{2}{\ln 3}\log_3 t + \frac{2}{\ln^2 3}\right) + C$       77.  $\frac{1}{2}e^x(\sin x - \cos x) + C$
78.  $\frac{1}{\ln^2 3+1}3^x(\sin x + \ln 3 \cos x) + C$       79.  $\frac{1}{2}e^{-x}(\sin x - \cos x) + C$
80.  $3\ln|x| + C$       81.  $\frac{1}{4x^4} + C$       82.  $\frac{5}{2}\ln|2x-3| + C$       83.  $-\frac{2}{9(1-3x)^3} + C$
84.  $\frac{1}{\sqrt{2}}\arctan\frac{x-1}{\sqrt{2}} + C$       85.  $\frac{1}{2}\ln(x^2 + 4x + 5) - 2\arctan(x+2) + C$

$$86. -\ln(x^2 + 3) + \frac{4}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + C \quad 87. -\frac{1}{2} \ln(x^2 - 2x + 5) - \frac{3}{2} \arctan \frac{x-1}{2} + C$$

$$88. -\frac{1}{2(x^2+1)} + C$$

$$89. \frac{x}{2(x^2+1)} + \frac{1}{2} \arctan x + C$$

$$90. 5 \ln|x + 5| + \ln|x - 1| + C$$

$$91. 2 \ln|x + 3| - \ln|x - 1| + C$$

$$92. \ln|x + 4| + 2 \ln|x - 3| - \frac{4}{x-3} + C$$

$$93. \frac{x-1}{(x-2)^2} + C$$

$$94. \ln|x + 1| - \arctan \frac{x}{3} + C$$

$$95. \ln \left| \frac{x+1}{x} \right| - \frac{1}{x} + C$$

$$96. \frac{1}{2} \ln|x| - 2 \ln|x + 2| + \frac{3}{2} \ln|x - 2| + C$$

$$97. -\frac{3}{4} \ln|x| + \frac{1}{2} \ln|x - 2| + \frac{1}{4} \ln|x + 4| + C$$

$$98. \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \arctan x + C$$

$$99. \frac{1}{2} \ln(x^2 + 1) + \frac{1}{2(x^2+1)} + C$$

$$100. \frac{1}{4} x^4 - x^3 - \ln|x + 3| + C$$

$$101. \frac{2}{3} x^3 + 3x + 3 \ln \left| \frac{x-2}{x+2} \right| + C$$

$$102. \frac{1}{2} x^2 + \ln|x^2 + x - 2| + C$$

$$103. x - 4 \arctan \frac{x}{4} + C$$

$$104. x^2 - 6x - 3 \ln|x - 1| + 12 \ln|x + 4| + C$$

$$105. x + \ln(x^4 - x^3 - x + 1) + C$$

$$106. \frac{1}{2} x^2 - x - 4 \ln|x| + 2 \arctan \frac{x}{2} + C$$

$$107. x + 2 \ln|x| - \arctan(x - 1) + C$$

$$108. \frac{1}{3} x^3 + \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \arctan x + C$$

$$109. \frac{1}{2} x^2 - \frac{1}{2(x^2+1)} - \ln(x^2 + 1) + C$$

$$110. 2\sqrt{x} - 2 \ln(\sqrt{x} + 1) + C$$

$$111. 2\sqrt{x-1} - 2 \arctan \sqrt{x-1} + C$$

$$112. \frac{3}{4} (x-3) \sqrt[3]{x+1} + C \quad 113. 6\sqrt[6]{x} + 3 \ln \left| \frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1} \right| + C \quad 114. -\frac{1}{3} \sqrt{\left( \frac{x+2}{x} \right)^3} + C$$

$$115. -2 \sqrt{\frac{x+2}{x}} + \ln \left| \frac{\sqrt{x+2} + \sqrt{x}}{\sqrt{x+2} - \sqrt{x}} \right| + C$$

$$116. -\ln \left| \frac{1 + \sqrt{1-x^2}}{x} \right| + C$$

$$117. \sqrt{1-x^2} - \ln \left| \frac{1 + \sqrt{1-x^2}}{x} \right| + C$$

$$118. \frac{1}{4} \sin 2x - \frac{1}{12} \sin 6x + C$$



119.  $\frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x + C$

120.  $\frac{1}{2} \sin x + \frac{1}{14} \sin 7x + C$

121.  $\frac{1}{2} x + \frac{1}{20} \sin 10x + C$

122.  $-\frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x + \frac{1}{24} \cos 6x + C$

123.  $\frac{1}{2} \sin x - \frac{1}{12} \sin 3x - \frac{1}{20} \sin 5x + C$

124.  $\frac{1}{\cos x} + C$

125.  $-\frac{1}{2 \sin^2 x} + C$

126.  $\frac{1}{(\cos x + 3)^3} + C$

127.  $\frac{1}{2} \arctan\left(\frac{1}{2} \sin x\right) + C$

128.  $\frac{1}{3} \tan^3 x + C$

129.  $\cot^3 x - \cot x + C$

130.  $\ln|\tan x| + C$

131.  $\frac{1}{2} \arctan(2 \tan x) + C$

132.  $-\frac{2}{\tan^2 x + 1} + C$

133.  $\ln \left| \frac{\tan \frac{x}{2} + 1}{\tan \frac{x}{2} - 1} \right| = \ln \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right| + C$

134.  $x - \tan \frac{x}{2} + C$

135.  $\frac{1}{5} \ln \left| \frac{2 \tan \frac{x}{2} - 1}{\tan \frac{x}{2} + 2} \right| + C$

136.  $\frac{2}{3} \sqrt{x^3} + C$

137.  $\frac{2^{x+2} 3^{-x-3}}{\ln 2 - \ln 3} + C$

138.  $\frac{3}{2} \sqrt[3]{(x-3)^2} + C$

139.  $\frac{4}{9} \sqrt[4]{(x^3-8)^3} + C$

140.  $\frac{1}{6} \ln \left| \frac{x+5}{x-1} \right| + C$

141.  $\ln(x^2 - 2x + 2) + \arctan(x - 1) + C$

142.  $-4 \ln|x| + 2 \ln(x^2 + 1) + 3 \arctan x + C$

143.  $x + \ln \left| 1 - \frac{1}{x^2} \right| + C$

144.  $(x^3 + x) \arctan x - \frac{1}{2} x^2 + C$

145.  $x \arcsin 2x + \frac{1}{2} \sqrt{1 - 4x^2} + C$

146.  $\frac{1}{2} x^2 \log 4x - \frac{1}{4 \ln 10} x^2 + C$

147.  $\frac{1}{\ln^2 2 + 1} 2^x (\ln 2 \sin x - \cos x) + C$

148.  $4 \sqrt[4]{x^3} + 2 \sqrt{x} + C$

149.  $\sqrt{x^2 - 1} - \ln|x + \sqrt{x^2 - 1}| + C$

150.  $\frac{1 - \cos x}{\sin x} + C$

151.  $\frac{1 + \sin 2x}{2 \cos^2 x} + C$

152.  $\frac{15}{4}$       153.  $\frac{5}{12}$       154. 1      155. -1      156.  $\frac{\pi}{3}$   
 157.  $\frac{8}{9\ln 3}$       158.  $-\frac{24}{5}$       159.  $\frac{3}{2}$       160.  $\frac{116}{5}$       161. -43  
 162. 25      163.  $-\frac{49}{30}$       164.  $\frac{26}{3}$       165.  $\frac{15}{4}$       166.  $-\frac{\sqrt{2}}{12}$   
 167.  $\frac{3}{8}$       168.  $3\ln 5 - 4$       169.  $\frac{1}{2}$       170.  $\frac{16}{3} - 2\ln 3$
171.  $-\frac{2}{3}$       172.  $\frac{\pi}{2}$       173. -2      174.  $\frac{e^2 + 1}{4}$   
 175.  $\frac{e^2 - 5}{e}$       176.  $\frac{9 - \ln 10}{10\ln 10}$       177.  $\frac{\pi - 2}{4}$       178.  $\frac{8\ln 2 - 4}{\ln 2}$   
 179.  $\frac{\pi}{4}$       180.  $\frac{5 - 8\ln 2}{4}$       181.  $\frac{\pi - 2\ln 2}{12}$       182.  $-\frac{e^\pi + 1}{2}$   
 183. 0      184.  $-\frac{3}{2}$       185.  $\frac{146}{15}$       186. 0  
 187.  $\pi$       188. 0      189.  $\frac{16}{3}$       190.  $\frac{128}{3}$
191.  $f(x) = \frac{2}{3}\sqrt{x^3}$ ,  $f(4) = \frac{16}{3}$       192.  $f(x) = \frac{1}{3}x^3 + x^2 - \frac{2}{3}$ ,  $f(0) = -\frac{2}{3}$   
 193.  $f(x) = \ln 2 - \ln|x|$ ,  $f(-e) = \ln 2 - 1$   
 194.  $f(x) = 4 + x - \frac{3}{4}\sqrt[3]{x^4}$ ,  $f(1) = \frac{17}{4}$   
 195.  $f(x) = \frac{3x^3 - 1}{2x}$ ,  $f(2) = \frac{23}{4}$   
 196.  $f(x) = \ln\left|\frac{x+1}{x+2}\right| + 1$ ,  $f(3) = \ln\frac{4}{5} + 1$
197. 9      198. 4      199. 24      200. 1      201. 2  
 202.  $\frac{9}{2}$       203.  $\frac{38}{3}$       204. 1      205. 16      206. 24

$$207. \quad P = \int_{-1}^2 (2 + y - y^2) dy = \frac{9}{2} \quad 208. \quad P = \int_0^1 (y-1)^2 dy = \frac{1}{3}$$

$$209. \quad P = 2 \int_1^2 (-x^2 + 3x - 2) dx = \frac{1}{3} \quad 210. \quad P = 4 \int_0^2 (4 - x^2) dx = \frac{64}{3}$$

$$211. \quad \frac{64}{3} \quad 212. \quad \frac{16}{3} \quad 213. \quad \frac{3}{4} \quad 214. \quad P = 2 \int_0^2 (4x - x^3) dx = 8$$

$$215. \quad P = \int_0^1 (x^3 + \sqrt{x}) dx + \int_1^4 (2 - x + \sqrt{x}) dx = \frac{49}{12} \quad 216. \quad \frac{7}{3} \quad 217. \quad \frac{4}{3}$$

$$218. \quad \frac{1}{3} \quad 219. \quad \frac{1}{12}$$

$$220. \quad \frac{1}{2} P = \int_{-3}^1 \sqrt{x+3} dx - \int_0^1 \sqrt{4x} dx = \int_0^2 \left(3 - \frac{3}{4} y^2\right) dy = 4, \quad P = 8$$

$$221. \quad P = \int_{-2}^2 \sqrt{x+2} dx - \int_0^2 \sqrt{2x} dx = \int_0^2 \left(2 - \frac{1}{2} y^2\right) dy = \frac{8}{3}$$

$$222. \quad P = \int_0^4 \sqrt{x} dx + \int_4^6 (6-x) dx = \int_0^2 (6-y-y^2) dy = \frac{22}{3}$$

$$223. \quad P = \int_{\frac{1}{2}}^3 \left(\frac{10}{3} - \frac{2}{3}x - \frac{x+1}{x}\right) dx = \int_{\frac{4}{3}}^3 \left(5 - \frac{3}{2}y - \frac{1}{y-1}\right) dy = \frac{35}{12} - \ln 6$$

$$224. \quad P = \int_0^1 \left(\frac{\pi}{2} - \operatorname{arctg} x\right) dx = \int_0^{\frac{\pi}{4}} \operatorname{tg} y dy + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} dy = \frac{\ln 4 + \pi}{4}$$

$$225. \quad P = \int_0^e dx - \int_1^e \ln x dx = \int_0^1 e^y dy = e - 1$$

$$226. \quad \frac{32\pi}{5} \quad 227. \quad \frac{625\pi}{6} \quad 228. \quad \frac{256\pi}{5} \quad 229. \quad \frac{49\pi}{30} \quad 230. \quad \frac{162}{5} \quad 231. \quad \frac{\pi}{2}$$

$$232. \quad V = \pi \int_0^1 x dx + \pi \int_1^4 \left(\frac{4-x}{3}\right)^2 dx = \frac{3\pi}{2}$$

$$233. \quad V = \pi \int_{\frac{1}{8}}^1 x^{\frac{2}{3}} dx + \int_1^2 x^{-2} dx - \pi \int_{\frac{1}{8}}^2 \frac{1}{4} dx = \frac{49\pi}{80}$$

$$234. \quad V = \pi \int_{-\frac{1}{2}}^4 (2x+1) dx - \pi \int_1^4 (x-1)^2 dx = \frac{45\pi}{4}$$

$$235. \quad V = \pi \int_{-1}^3 (-x^2 + 2x + 6)^2 dx - 2\pi \int_{-1}^0 (x^2 - 2x)^2 dx = \frac{1936\pi}{15}$$

236.  $\frac{16\pi}{15}$

237.  $\frac{\pi}{2}$

238.  $8\pi$

239.  $\frac{3\pi}{10}$

240.  $\frac{34\pi}{5}$

$$241. \quad V = 8\pi \int_0^{\frac{1}{2}} y dy + \pi \int_{\frac{1}{2}}^1 \frac{1}{y^2} dy - \pi \int_0^1 \sqrt[3]{y^2} dy = \frac{7\pi}{5}$$

242.  $\frac{38}{3}$

243.  $\frac{19}{27}$

244.  $\ln(1+\sqrt{2}) + \sqrt{2}$

245.  $\frac{1}{2} \ln(2+\sqrt{5}) + \sqrt{5}$

246.  $\ln 3 - \frac{1}{2}$

247.  $\ln \frac{9}{8} + 4$

248.  $l = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1 + \frac{\cos^2 y}{\sin^2 y}} dy = \frac{1}{2} \ln 3$

249.  $l = \int_{\ln \frac{\sqrt{3}}{2}}^{\ln \frac{2\sqrt{2}}{3}} \sqrt{\frac{1}{1-e^{2x}}} dx = \frac{1}{2} \ln \frac{3}{2}$

250.  $\frac{7\pi}{9}$

251.  $\frac{56\pi}{3}$

252.  $\frac{6\pi}{5}$

$$253. \quad P = 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{2}{\sin x} \sqrt{1 + \frac{4 \cos^2 x}{\sin^4 x}} dx = 4\sqrt{2}\pi$$

254.  $2\pi$

255.  $\frac{13\pi}{3}$

$$256. \quad P = \frac{\pi}{3} \int_0^2 y \sqrt{y^2 + 4y} dy = \frac{4\pi \ln(2 + \sqrt{3})}{3}$$

257. 0,165

258. -13,811

259. 3,093

260. 0,342

261. 8,018

262. 223,054

263. 1,585

264. -16,083

265. -0,439

266. 0,195

267. 2,063

268. 268,383

269. 21,998

270. 4,554

271. 0,334

272. -0,732

273. 0,000

274. -87,786

275. 0,635

276. -4,022

$$277. \frac{64}{5} \quad 278. \frac{1793}{12} \quad 279. -\frac{1}{3} \quad 280. 0$$

$$281. \int_{-3}^0 (-x) dx + \int_0^2 x dx = \frac{13}{2} \quad 282. \frac{5}{2} \quad 283. \frac{1}{2} \quad 284. \frac{1}{2}$$

$$285. \int_{-3}^0 |x+2| dx = \int_{-3}^{-2} (-x-2) dx + \int_{-2}^0 (x+2) dx = \frac{5}{2}$$

$$286. 5 \quad 287. 2 \quad 288. 0 \quad 289. e^3 + e - 2 \quad 290. 1$$

$$291. \quad P = 4 \int_0^a \sqrt{a^2 - x^2} dx = \pi a^2 \quad l = 4a \int_0^a \frac{1}{\sqrt{a^2 - x^2}} dx = 2\pi a$$

$$292. V = \pi \frac{a^2}{v^2} \int_0^v x^2 dx = \frac{\pi}{3} a^2 v \quad P = \pi a^2 + 2\pi \frac{a\sqrt{a^2 + v^2}}{v^2} \int_0^v x dx = \pi a \left( a + \sqrt{a^2 + v^2} \right)$$

$$293. V = 2\pi \int_0^a (a^2 - x^2) dx = \frac{4\pi}{3} a^3 \quad P = 4\pi a \int_0^a dx = 4\pi a^2$$

294. Promatraj vrtnju kružnice  $x^2 + (y-b)^2 = a^2$  oko osi  $x$

$$V = 8\pi b \int_0^a \sqrt{a^2 - x^2} dx = 2\pi^2 a^2 b \quad P = 8\pi ab \int_0^a \frac{1}{\sqrt{a^2 - x^2}} dx = 4\pi^2 ab$$

- 295.** Jest                      **296.** Nije                      **297.** Jesu                      **298.** Nisu  
**299.** Jest                      **300.** Nije                      **301.** Jest                      **302.** Jesu  
**303.** Jest                      **304.** Jest                      **305.** Nije                      **306.** Jesu
- 307.**  $y = x^2 - \sin x + C$                       **308.**  $y = x - e^x + C$   
**309.**  $y = \ln|Cx|$                       **310.**  $y = \ln^2 x + \ln x + C$   
**311.**  $y = x^3 + x^2 + C_1x + C_2$                       **312.**  $y = \frac{1}{6}x^3 + \ln|x| + C_1x + C_2$   
**313.**  $y = e^x + C_1x - \ln|C_2x|$                       **314.**  $y = \cos x + x \sin x + C_1x + C_2$   
**315.**  $y = C_1x^2 + C_2x + C_3$                       **316.**  $y = \frac{1}{2}x^3 - \sin x + C_1x^2 + C_2x + C_3$   
**317.**  $y = -xe^x + C_1x^2 + C_2x + C_3$   
**318.**  $y = \ln^2 x + 3 \ln x + C_1x^2 + C_2x + C_3$
- 319.**  $y = x^3 + x^2 - \ln|x| + 1$                       **320.**  $y = \sin x + \cos x - 3$   
**321.**  $y = e^{5x} - 5x^2 + 4$                       **322.**  $y = x^4 - x^2 - 11x + 12$   
**323.**  $y = \frac{x-1}{x^2} - \frac{1}{4}x + \frac{13}{16}$                       **324.**  $y = \tan x + \cot x + x - 2$   
**325.**  $y = e^x - x^2$                       **326.**  $y = x^3 + \frac{1}{x^3} - 6x^2 + 20x - 13$   
**327.**  $8\sqrt{x^3} - 3y^4 = C$                       **328.**  $x = \sqrt{y^2 - 1} + C$   
**329.**  $y = \frac{1}{2} \ln^2 x + C$                       **330.**  $\frac{x^2}{2} + \frac{y^3}{3} = C$

331.  $x^2 - y^2 - 2(\cos x + \sin y) = C$

333.  $\frac{1}{x} - \frac{1}{y} = C$

335.  $x^2 + y^2 = C$

337.  $x = \ln(y^2 + 3) + C$

339.  $y = -x \ln|Cx|$

341.  $x + y \ln|Cy| = 0$

343.  $y = x \ln|Cx|$

345.  $y^2 = 2x^2 \ln|Cx|$

347.  $x = y \ln|Cy|$

349.  $\ln|Cx| = \tan \frac{y}{x}$

351.  $y = Ce^{-3x}$

353.  $y = Ce^{\cos x}$

355.  $y = (x^2 + C)e^x$

357.  $y = x^2 + x + \frac{C}{x}$

359.  $y = (x + C)e^{x^2}$

361.  $y = x^4 + \frac{C}{x}$

363.  $y = (x + C)e^{\frac{1}{x}}$

332.  $(x-1)e^x - ye^y = C$

334.  $y = Ce^{x^2}$

336.  $\sqrt{x^3} - 3\sqrt{y} = C$

338.  $e^{-x} + e^y = C$

340.  $x^2 = 2y^2 \ln|Cx|$

342.  $\ln|Cx| = \frac{1}{2} \ln^2 \frac{x}{y}$

344.  $y = -\frac{x}{\ln|Cx|}$

346.  $\ln|Cx| + \cos \frac{y}{x} = 0$

348.  $x^2 + 2y^2 \ln|Cx| = 0$

350.  $y = 2x \arctan Cx$

352.  $y = Cx$

354.  $y = Cx^x$

356.  $y = Ce^{-2x} + 2x^2 - 2x + 1$

358.  $y = \sin x + \frac{\cos x}{x} + \frac{C}{x}$

360.  $y = Ce^{\cos x} + 1$

362.  $y = x(\ln^2 x + C)$

364.  $y = Ce^{tgx} - 1$

365.  $y = 4e^x + 1$
367.  $y = e^{\sin x}$
369.  $y = -\frac{e^x}{e^x + C}$
371.  $y = \pm e^{2x} \sqrt{x + C}$
373.  $y = \pm \frac{e^{-x^2}}{\sqrt{e^{-2x^2} + C}}$
375.  $y = C_1 + C_2 e^{2x} - e^x$
377.  $y = \ln^3 x + C_1 \ln x + C_2$
379.  $y = C_1 e^{2x} + C_2 e^{-3x}$
381.  $y = (C_1 x + C_2) e^{2x} + 1$
383.  $y = C_1 \sin 5x + C_2 \cos 5x$
385.  $y = (x^2 + C_1 x + C_2) e^x$
387.  $y = C_1 + (C_2 - \frac{1}{2}x) e^{-2x}$
389.  $y = C_1 \sin x + (C_2 - \frac{1}{2}x) \cos x$
366.  $y = \frac{1}{3}x^2 + \frac{2}{3x}$
368.  $y = -1$
370.  $y = x(x + C)^2$
372.  $y = \sqrt[3]{\frac{3}{5}x^2 + \frac{C}{x^3}}$
374.  $y = e^x (e^{\frac{1}{3}x} + C)^3$
376.  $y = C_1 + C_2 x^2 + x^3$
378.  $y = \cos^2 x + C_1 \cos x + C_2$
380.  $y = C_1 e^{4x} + C_2$
382.  $y = (C_1 x + C_2) e^{\frac{1}{2}x}$
384.  $y = (C_1 \sin 3x + C_2 \cos 3x) e^{-2x}$
386.  $y = C_1 \sin x + C_2 \cos x + x$
388.  $y = C_1 e^x + (2x^2 - 2x + C_2) e^{3x}$
390.  $y = C_1 e^x + C_2 e^{2x} + \cos x$
391.  $y = (C_1 - x) \sin 2x + (C_2 + x) \cos 2x$
392.  $y = C_1 + C_2 e^x - \frac{1}{2}(\sin x + \cos x) e^x$
393.  $y = (\frac{1}{4}x^2 - \frac{1}{2}x + C_1) e^x + C_2 e^{-x}$
394.  $y = C_1 + C_2 e^{-x} + (e^{-x} + 1) \ln(e^x + 1)$



395.  $y = x^2 - 3x + 4$

397.  $y = (\sin x + \frac{1}{3})e^{3x}$

399.  $y = \frac{1}{2} \ln|x| + C_1x^2 + C_2x + C_3$

401.  $x + y = C$

403.  $x = y \ln|Cy|$

405.  $y = Cx + x \cos x$

407.  $y = (C_1 \sin 4x + C_2 \cos 4x)e^{3x}$

409.  $y = C_1 + C_2e^{2x} - x^2 + 3x$

411.  $y = C_1e^x + C_2e^{2x} + C_3$

413.  $y = C_1e^x + C_2e^{-x} + C_3x + C_4$

396.  $y = e^{2x} - e^x$

398.  $y = e^x + C_1x + C_2$

400.  $y = e^x + C_1x + C_2$

402.  $xy = C$

404.  $y = x - \frac{C}{x}$

406.  $y = (Ce^x + x + 1)^2$

408.  $y = (C_1x + C_2)e^{2x} + 1$

410.  $y = C_1e^x + (C_2 + 4 \sin x + 5 \cos x)e^{\frac{1}{5}x}$

412.  $y = x + C_1 \sin x + C_2 \cos x + C_3$

414.  $y = C_1x + C_2x^2 + C_3e^{3x} + C_4$